## 1.2 <br> Patterns in a Table of Values

## Focus on...

After this lesson, you will be able to...
$\square$ identify relationships in a table of values
$\square$ decide if a table of values represents a linear relation
$\square$ graph points represented by values in a table

## Materials

- grid paper


As humans, we see patterns all around us. It is a way for us to make sense of our world. However, different people may see patterns in different places and in different ways. In the picture, Kendra describes a pattern she sees in the wallpaper. What patterns can you identify?

## Explore the Math

How can you represent patterns?


1. What are three patterns you see in the design?
2. How many squares and triangles are in this design?
3. The design can be lengthened or shortened. Each section of the design must have only complete squares and triangles. If this design has ten sections, what does one section of the design look like?
4. Make a table of values like the one shown. Complete your table for one to four sections in the design.

| Number of <br> Sections | Number of <br> Vertical <br> Segments | Number of <br> Horizontal <br> Segments | Number of <br> Slanting <br> Segments |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |

5. a) Use words to describe the relationship between the number of sections and the number of vertical segments.
b) Choose variables to represent the number of sections and the number of vertical segments. Why did you choose these letters?
c) Draw a graph to show the relationship between the number of sections and the number of vertical segments. Label the axes with the variables you chose in part b), and with the titles Number of Sections and Number of Vertical Segments.
d) Describe in words the relationship between the variables.
e) What is an expression for the number of vertical segments in terms of the number of sections?
6. a) Draw a graph to show the relationship between the number of sections in the design and the data in one of the other columns in the table. Label each axis with a variable and a title.
b) What is an expression that represents the data in the column you chose in terms of the number of sections in the design?

## Reflect on Your Findings

7. a) What ways did you use to represent patterns in this activity?
b) Which parts of the activity did you find most challenging? Why?

## WWW Web Link

To practise your graphing skills, go to
www.mathlinks8.ca and follow the links.

## Literacy \& Link

A variable is a letter that represents an unknown quantity. For example, in $3 a-5$, the variable is $a$.

It can be helpful to choose variables that are meaningful. For example, $t$ for time and $s$ for score.

## Literacy 8 Link

An expression is any single number or variable, or a combination of operations (,,$+- \times$, $\div$ ) involving numbers and variables. An expression does not include an equal sign. The following are examples of expressions:
$5 \quad r \quad 8 t$ $x+9 \quad 2 y-7$

## Example 1: Identify the Relationship in a Table of Values

The pattern in the table of values represents a linear relation.
a) Graph the ordered pairs in the table of values.
b) What is the difference in value for consecutive $A$-values? for consecutive

| $\boldsymbol{A}$ | $\boldsymbol{B}$ |
| :---: | ---: |
| 0 | 0 |
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 | $B$-values?

c) Describe in words the relationship between the values for $A$ and $B$.
d) What is an expression for $B$ in terms of $A$ ?

## Solution


b) The difference in consecutive $A$-values is 1 . Find the difference in consecutive $B$-values.

$$
\begin{array}{ll}
3-0=3 & 9-6=3 \\
6-3=3 & 12-9=3
\end{array}
$$

The difference in consecutive $B$-values is 3 .
c) When $A$ increases by $1, B$ increases by 3 .
d) Use the difference in $B$-values of 3 . Each $B$-coordinate is 3 times its $A$-coordinate.


An expression for $B$ is $3 \times A$, or $3 A$.

## Show You Know

The table of values represents a linear relation.

| $\boldsymbol{x}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\boldsymbol{y}$ | 5 | 10 | 15 | 20 | 25 | 30 | 35 |

a) Graph the ordered pairs in the table of values.
b) What is the difference in value for consecutive $x$-values? What is the difference in value for consecutive $y$-values?
c) Describe in words the relationship between the $x$-values and $y$-values.
d) What is an expression for $y$ in terms of $x$ ?

## Example 2: Use a Table to Determine a Linear Relation

For each table of values below, answer the following questions:
a) What is the pattern in the values for the first variable in each table?
b) What is the difference in consecutive values for the second variable in each table? Is the difference within each table the same?
c) Graph each set of ordered pairs. Which relations are linear?

Table 1

| $\boldsymbol{x}$ | 2 | 4 | 6 | 8 |
| :--- | :--- | :--- | ---: | ---: |
| $\boldsymbol{y}$ | 3 | 7 | 11 | 15 |

Table 2

| $\boldsymbol{m}$ | 1 | 2 | 3 | 4 |
| :---: | :--- | :--- | :--- | :--- |
| $\boldsymbol{n}$ | 1 | 4 | 7 | 8 |

d) How does your answer in part c) compare with your answer in part b)?

## Solution

a) Table 1: The $x$-coordinates differ by 2 .

Table 2: The $m$-coordinates differ by 1.
The consecutive values of the first variables differ by the same amount within each table.
b) Table 1: The $y$-coordinates differ by 4 . The difference is the same.

Table 2: The $n$-coordinates differ by 3,3 , and 1 . The difference is not the same.
c)


The relation is linear.

The relation is not linear.
d) Table 1: The difference in the $y$-coordinates is the same and the relation is linear.
Table 2: The difference in the $n$-coordinates is not the same and the relation is not linear.

## Show You Know

Determine whether each table represents a linear relation. Explain how you know. Check each answer by graphing the ordered pairs.

## Table 1

Table 2

| $\boldsymbol{p}$ | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{q}$ | 0 | 1 | 3 | 6 |


| $\boldsymbol{A}$ | 1 | 4 | 7 |
| :--- | :--- | ---: | ---: |
| $\boldsymbol{B}$ | 1 | 10 | 19 |

## Literacy 8 Link

If consecutive values for the first variable do not have the same difference, it is difficult to tell from the table whether the relationship is linear. You may be able to tell by drawing a graph.

## Example 3: Use a Table of Values in Solving a Problem

Photo World charges $\$ 3$ for the first enlargement and $\$ 2$ for each additional enlargement.
a) Make a table of values showing the cost in relation to the number of enlargements for one to five enlargements.
b) Is this a linear relation? Why?
c) What is an expression for the cost in relation to the number of enlargements?
d) What is the cost of 15 enlargements?


## Solution

a)

| Number of Enlargements, $\boldsymbol{n}$ | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | ---: |
| Cost, C (\$) | 3 | 5 | 7 | 9 | 11 |

b) The difference between consecutive numbers of enlargements is 1 . Determine the difference for consecutive costs.
$5-3=2$
$7-5=2$
$9-7=2$
$11-9=2$
Consecutive costs differ by 2 .
Each consecutive number of enlargements differs by the same value.
Each consecutive cost differs by the same value.
The relation is linear.
c) The cost increases by $\$ 2$ for each additional enlargement.

Let $n$ represent the number of enlargements.
$2 n$ will be part of an expression for the cost.
Check using a value for $n$ from the table: $n=4$.
$2(4)=8$
The value of 8 is not the same as the value of 9 in the table.
You need to add 1 to get from 8 to 9 .
An expression for the cost is $2 n+1$, where $n$ is the number of enlargements.
d) Substitute $n=15$ into the expression $2 n+1$.
$2(15)+1=31$
The cost of 15 enlargements is $\$ 31$.

## Show You Know

Sky sells magazine subscriptions. She receives $\$ 20$ for every five subscriptions she sells. The table shows the relationship between the number of subscriptions she sells and the pay she receives.

| Number of Subscriptions, $\boldsymbol{n}$ | 0 | 5 | 10 | 15 | 20 | 25 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Pay, $\mathbf{P}(\mathbf{\$})$ | 0 | 20 | 40 | 60 |  |  |

a) Copy and complete the table.
b) Is this a linear relation? How can you tell?
c) What is an expression for Sky's pay in relation to the number of subscriptions she sells?
d) How much does Sky receive for selling 40 subscriptions?

## Key Ideas

- The following are some ways that you can represent a linear relation.
- Table of values:

| Mass, $\boldsymbol{m}$ (g) | 0 | 100 | 200 | 300 |
| :--- | :--- | :--- | :--- | :--- |
| Cost, $\boldsymbol{C}$ ( (¢) | 0 | 200 | 400 | 600 |

- Words: The cost in cents is 2 times the mass in grams.
- Ordered pair: ( $m, 2 m$ )
- Expression: The cost in cents is $2 m$, where $m$ is the mass in grams.
- Graph:

- You can sometimes tell from a table whether the relationship is linear.

Table 1

| $\boldsymbol{p}$ | 2 | 3 | 4 | 5 |
| :--- | :--- | ---: | ---: | ---: |
| $\boldsymbol{q}$ | 7 | 13 | 19 | 25 |

Table 2

| $\boldsymbol{p}$ | $\boldsymbol{q}$ |
| :---: | :---: |
| 20 | 31 |
| 40 | 27 |
| 60 | 23 |
| 80 | 19 |

You can tell that the relationships in the above tables are linear because both of the following statements are true:

- Each consecutive value for $p$ changes by the same amount.
" Each consecutive value for $q$ changes by the same amount.


## Communicate the Ideas

1. Giselle and Tim are discussing the table of values shown.

| $\boldsymbol{m}$ | 3 | 5 | 7 | 9 |
| :---: | :--- | :--- | :--- | :--- |
| $\boldsymbol{a}$ | 1 | 3 | 5 | 7 |

Who is correct? How do you know?
2. a) Describe a real-life situation that matches the pattern in the table of values.

| $\boldsymbol{n}$ | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{p}$ | 1 | 3 | 5 | 7 |

b) What do $n$ and $p$ represent in your situation?

3. You are given a table of values. Can you sometimes, always, or never tell whether the relationship is linear without drawing the graph? Use examples to support your answer.

## BnEb Your Inderstandng

## Practise

For help with \#4 to \#7, refer to Example 1 on page 344.
4. Graph the ordered pairs in the table of values.

| $\boldsymbol{a}$ | $\boldsymbol{d}$ |
| :---: | ---: |
| 1 | 5 |
| 2 | 8 |
| 3 | 11 |
| 4 | 14 |

5. Draw a graph using the ordered pairs in the table of values.

| $\boldsymbol{w}$ | $\boldsymbol{t}$ |
| ---: | ---: |
| 1 | 1 |
| 4 | 7 |
| 7 | 13 |
| 10 | 19 |

7. The table of values shows a linear relation.

| $\boldsymbol{n}$ | 3 | 4 | 5 | 6 | 7 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| $\boldsymbol{d}$ | 18 | 24 | 30 | 36 | 42 |

a) Graph the ordered pairs.
b) What is the difference in value for consecutive $n$-values? What is the difference in value for consecutive $d$-values?
c) In words, describe the relationship between $n$ and $d$.
d) What is an expression for $d$ in terms of $n$ ?

For help with \#8 and \#9, refer to Example 2 on page 345.
8. For each table of values, tell whether the relationship is linear. Explain your answer. Check by graphing the ordered pairs.
a)

| $\boldsymbol{c}$ | $\boldsymbol{d}$ |
| :---: | :---: |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |
| 5 | 16 |

b)

| $x$ | $y$ |
| :---: | ---: |
| 0 | -3 |
| 1 | 1 |
| 3 | 9 |
| 5 | 17 |

9. Tell whether the relationship for each table of values is linear. Explain how you know.
a)

| $\boldsymbol{p}$ | 4 | 7 | 10 | 16 |
| :---: | ---: | ---: | ---: | ---: |
| $\boldsymbol{q}$ | 11 | 17 | 23 | 29 |

b)

| $\boldsymbol{x}$ | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{y}$ | 3 | 2 | 1 | 0 |

For help with \#10 and \#11, refer to Example 3 on page 346.
10. Mara reads at a rate of 90 words per minute.
a) Make a table of values that shows the total number of words Mara reads in one to six minutes. Use whole minutes.
b) Is this a linear relation? Explain.
c) What is an expression for the number of words Mara reads in terms of time?
d) How many words does Mara read in 12 min ?
11. The dosage of a certain medication to be given to a child is related to the child's mass. A 10-kg child receives 50 mg of the medication. For each increase in mass of 1 kg , the child receives an additional 10 mg of the medication.
a) Make a table showing the dosage for children with an increase of 1 kg to 10 kg over the mass of 10 kg . Use whole kilograms only.
b) Is this a linear relation? Explain how you know.
c) What is an expression for the dosage in terms of the increase in mass over 10 kg ?
d) What is the dosage for a child with a mass of 27 kg ?
e) Can your table of values start at 0 kg ? Why or why not?

## Apply

12. Evan has $\$ 6$ in quarters and dimes.

a) Name five combinations of quarters and dimes that Evan might have.
b) Make a table of values showing the relationship between quarters and dimes. Include five pairs of values in your table.
c) Draw a graph. Is the relationship between quarters and dimes linear in this example? Explain.
d) What is the largest possible number of dimes? of quarters?
13. Divers experience an increase in pressure as they dive deeper below sea level. The table shows the relationship between depth in metres and pressure in atmospheres.

| Depth (m) | Pressure (atm) |
| :---: | :---: |
| 0 | 1 |
| 10 | 2 |
| 20 | 3 |
|  | 4 |
|  | 5 |
|  | 6 |

a) Copy and complete the table.
b) Draw a graph for the ordered pairs in the table. How should your axes be labelled?
c) Divers sometimes become dizzy when the pressure exceeds 5 atm . Below what depth do divers tend to become dizzy?
14. The following pattern continues.


Figure 1


Figure 2


Figure 3
a) Make a table of values showing the figure number and the number of squares for the first six figures.
b) Write an expression showing the number of squares in terms of the figure number. What does your variable represent?
c) How many squares would appear in Figure 20?
d) How many more squares are in Figure 20 than in Figure 10? Show two ways to find the answer.
15. The following pattern of squares continues.
a) Copy and complete the table of values below that shows the relationship between the number of squares and the perimeter of each figure.

| Number of Squares | 1 | 2 |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Perimeter (cm) | 4 | 6 |  |  |

b) Draw a graph from the table of values.
c) Describe the patterns on the graph.
d) What is an expression for the perimeter in terms of the number of squares?
e) If the pattern continues, what is the perimeter when there are 50 squares?
16. As you climb a mountain, the temperature drops $1^{\circ} \mathrm{C}$ for every 150 m of increased height.
a) Copy and complete the table to show the relationship between height and temperature if the temperature at the bottom of the mountain is $20^{\circ} \mathrm{C}$.

| Height (m) | 0 | 150 | 300 | 450 | 600 | 750 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature <br> $\left({ }^{\circ} \mathbf{C}\right)$ | 20 |  |  |  |  |  |

b) Graph the ordered pairs
c) Is the relationship linear?
d) How high have you climbed if the temperature is $13{ }^{\circ} \mathrm{C}$ ?
17. A skydiver jumps from an airplane. The table provides data for the period of time shortly after the parachute opens, relating time in seconds to total distance descended in metres.

| Time (s) | Distance (m) |
| :---: | :---: |
| 10 | 300 |
| 11 | 354 |
| 12 | 408 |
| 13 | 462 |


a) Predict how a graph for these ordered pairs would look.
b) Graph the ordered pairs. Was your prediction correct?
c) Describe the graph in words.
18. A community centre has a new banquet hall. The centre charges $\$ 5$ per person to rent the hall.
a) Make a table of values showing the rental cost for $20,40,60,80$, and 100 people.
b) Graph the ordered pairs.
c) What is an expression for the rental cost in terms of the number of people?

## Extend

19. The community centre in $\# 18$ changes the cost for renting its banquet hall. The centre now charges $\$ 50$ plus $\$ 5$ per person.
a) Make a table of values showing the rental cost for 20, 40, 60, 80, and 100 people.
b) Graph the ordered pairs. How does the graph differ from the graph in \#18?
c) What is an expression for the rental cost in terms of the number or letter of people? What does each number or letter in your expression represent?
20. Jamal is renting snowboard equipment.

a) Copy and complete the table.

| Number of <br> Additional Days | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Rental Cost (\$) |  |  |  |  |  |  |

b) What is an expression for the rental cost in terms of the number of additional days?
c) What is the cost to rent the snowboard for a total of ten days? What might be a better option for Jamal instead of renting for ten days?

## MATH LINK

Have you ever gone for a canoe trip on the waters of one of Canada's national parks? You are planning a canoe trip. The cost to rent a canoe is $\$ 40$ a day. A national park pass for one week costs $\$ 36$.
a) Make a table of values showing the total cost for the pass and canoe for a trip from one to seven days.
b) Graph the ordered pairs in your table of values. Is this a linear relation? Explain.
c) What is an expression for the cost based on the number of days?
d) Think of a linear relationship to do with canoeing. Show the relationship using a table of values, a graph, words, and an expression.


