Applying Integer Operations

A famous submersible, the *Ben Franklin*, is at the Vancouver Maritime Museum. This submersible was built to study the currents and sea life along the east coast of North America in the Gulf Stream.

The *Ben Franklin* explored the Gulf Stream at depths from 200 m to 600 m. How would you represent these depths with integers? What operation would you use to find how many times as great a depth of 600 m is as a depth of 200 m?

Explore the Math

Why is it important to know the order of operations when solving problems involving integers?

Laura, Abeni, and Rob were discussing the following problem.

A submersible dives from the surface at 15 m/min for 6 min and then at 25 m/min for 20 min. What is the depth of the submersible after the dive?

They worked together to write the following expression to solve the problem.
$6 \times (-15) + 20 \times (-25)$

Then they evaluated the expression independently.
Laura evaluated the expression as $+1750$.
Abeni evaluated the expression as -590.
Rob evaluated the expression as -750.

1. Explain how the expression represents the problem.

2. Which student evaluated the expression correctly? How do you know?

3. What errors did the other two students make?

4. What is the depth of the submersible after the dive?

Reflect on Your Findings

5. Why is it important to know the order of operations when solving problems involving integers?

Example 1: Use the Order of Operations

Calculate.
a) $(-15) \div (-3) - (+4) \times (-2)$
b) $(-6) - (-9) + (-14) \div (+2)$
c) $-8 + (-2) \times [4 + (-1)]$

Solution

a) $(-15) \div (-3) - (+4) \times (-2)$ Multiply and divide in order, from left to right.
$= (+5) - (+4) \times (-2)$
$= (+5) - (-8)$ Subtract.
$= +13$

b) $(-6) - (-9) + (-14) \div (+2)$ Divide.
$= (-6) - (-9) + (-7)$ Add and subtract in order, from left to right.
$= (+3) + (-7)$
$= -4$

c) $-8 + (-2) \times [4 + (-1)]$ Brackets.
$= -8 + (-2) \times 3$ Multiply.
$= -8 + (-6)$ Add.
$= -14$

> Grouping is shown using square brackets, because -1 is already in round brackets.

Show You Know

Calculate.
a) $(+4) + (-7) \times (-3) - (+5)$
b) $(-16) \div [(+5) - (+6) + (-7)]$
c) $-2 \times [5 + (-3)] + (-15)$

Literacy Link

Omitting Positive Signs or Brackets

A positive integer can be written without the positive sign or brackets. For example, $(+3) \times (+4)$ can be written as 3×4.

Negative integers must include the negative sign. The brackets can be omitted from a negative integer that does not follow an operation symbol. For example, $(-9) \div (-3)$ can be written as $-9 \div (-3)$.

Example 2: Apply Integer Operations

One week in March in Peguis, Manitoba, the daily high temperatures were −2 °C, −6 °C, +1 °C, +2 °C, −5 °C, −8 °C, and +4 °C. What was the mean daily high temperature for that week?

Solution

The mean temperature is the sum of the temperatures divided by the number of temperatures.

$$\frac{(-2)+(-6)+(+1)+(+2)+(-5)+(-8)+(+4)}{7}$$

> The division bar is a division and grouping symbol, so you can rewrite the numerator inside brackets.

$$= [(-2) + (-6) + (+1) + (+2) + (-5) + (-8) + (+4)] \div 7 \quad \text{Brackets.}$$
$$= -14 \div 7 \quad \text{Divide.}$$
$$= -2$$

The mean daily high temperature for that week was −2 °C.

> You can omit the round brackets from both integers and the positive sign from +7.

Literacy Link

Understanding the Mean

The *mean* of a set of integers is found by dividing their sum by the number of integers. For example, the mean of −4, +8, and −10 is $\frac{(-4)+(+8)+(-10)}{3}$, which equals −2.

Show You Know

On four days in June in Resolute, Nunavut, the daily low temperatures were −6 °C, 0 °C, +1 °C, and −7 °C. What was the mean daily low temperature for those four days?

Key Ideas

- When solving a problem, you need to decide which operation(s) to perform on integers.
- Some integer problems involve the order of operations.
- The order of operations for integers is the same as for whole numbers and decimals.
  - Brackets.
  - Multiply and divide in order, from left to right.
  - Add and subtract in order, from left to right.

Communicate the Ideas

1. Lance evaluated the expression $-2 \times (4 + 5) + 3$ to equal 0.

a) What mistake did he make?

b) What is the correct value of the expression?

2. Ivan said that the mean of -18, -16, $+11$, and $+15$ is positive. Without calculating the value of the mean, Norah disagreed with him. How did she know that the mean is not positive?

3. If 15 times an integer is -255, would it be easier to determine the integer using multiplication or division? Explain.

Check Your Understanding

Practise

For help with #4 to #7, refer to Example 1 on page 313.

4. Calculate using the order of operations.
   a) $(+30) \div (-10) + (-20) \div (-1)$
   b) $(-2) \times [(+10) - (+8)] + (-7)$
   c) $(+6) + (+9) \times (-5) \div (-3)$

5. Calculate using the order of operations.
   a) $(-4) - (+8) \times (-2) - (+15)$
   b) $(-3) + (-18) \div (+2) \div (-3)$
   c) $(+16) \div [(+4) - (+2)] + (-4)$

6. Calculate.
   a) $(4 - 7) \times 2 + 12$
   b) $-10 \div 5 + 3 \times (-4)$
   c) $3 \times [14 + (-18)] - 8 \div (-4)$

7. Calculate.
   a) $-16 \div 2 \times (3 + 1)$
   b) $5 + (-9) \times 4 \div (-1)$
   c) $25 + (-10) - 3 \times [2 - (-2)]$

Apply

For help with #8 to #12, refer to Example 2 on page 314.

8. The daily low temperatures in Prince Rupert, British Columbia, for five days in January were -4 °C, $+1$ °C, -2 °C, $+1$ °C, and -6 °C. What is the mean of these temperatures?

9. The table shows changes in the number of subscribers to a community newsletter over a six-month period.

Month	Change in the Number of Subscribers
1	+8
2	+6
3	−12
4	+5
5	−9
6	−10

   a) What was the mean change per month in the number of subscribers?
   b) There were 207 subscribers at the beginning of this period. How many were there at the end?

10. Over a ten-year period, the population of Saskatchewan decreased from 989 000 to 979 000. What was the mean population change per year?

11. The mean of five integers is −11. What is the sum of the integers?

12. A golfer had a mean score of −3 for the 4 rounds of a golf tournament.

  a) What was the golfer's score for the whole tournament?

  b) If par for the course is 72 strokes, how many strokes did the golfer take to complete the tournament?

> ### Sports ⊖ Link
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> **Golf Scores**
> Par for a golf course is the total number of strokes an expert golfer should take to complete the course. An integer shows a golfer's performance for each round of golf. Using 68 strokes to complete a round on a par 70 course gives a score of 2 below par or −2 for that round. Using 74 strokes to complete a round on a par 70 course gives a score of 4 over par or +4 for that round.

13. The average temperature of Earth's surface is about 15 °C. The temperature of Earth's crust increases by about 25 °C for each kilometre below the surface. What is the average temperature 3 km below Earth's surface?

> **Did You Know?**
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> The high temperatures below Earth's surface can create hot springs. Water is heated underground and then runs to the surface before it can cool down. The water that feeds the Miette Hot Springs in Jasper National Park is at 54 °C when it reaches the surface. Hot springs support many forms of life. Dr. Kathleen Londry of the University of Manitoba studies an endangered species of snail that lives in the Cave and Basin Hot Springs in Banff National Park.

14. Ahmed had $100 in his savings account at the start of his summer job. For the next eight weeks, he added $70 to his savings each week. After he went back to school, he withdrew $55 per week from his savings. For how many weeks did he make withdrawals until his savings were gone?

15. A new freezer is at a room temperature of 22 °C. When the freezer is turned on, the temperature inside drops by 4 °C per hour. How long does it take the freezer to reach −10 °C?

16. A hang glider descends at 50 m/min for 3 min. The glider then catches an updraft and rises at 100 m/min for 2 min.

  a) What is the overall change in the hang glider's altitude over this 5-min period?

  b) What is the mean rate of change in the altitude over this period?

17. Because the batteries are low, Darren's watch is losing 9 min every hour. At 8:00 p.m., he sets the watch correctly. What time will his watch show when the correct time is 10:00 a.m. the next day?

18. a) A store that sells ski equipment lost a total of $18 000 in June, July, and August. What was the mean loss per month?

  b) The store broke even in April, May, and September. The store owner wants to make a profit of $54 000 for the year. To meet this target, what mean profit per month does the store need to make in the other six months?

19. Rohana earns $50 a week from babysitting. She spends $25, saves $15, and uses the rest to repay a loan of $100 from her sister.

  a) After six weeks, how much has Rohana spent, how much has she saved, and how much does she still owe her sister?

  b) How many more weeks will Rohana take to pay off the loan?

20. Write and evaluate an expression that represents each statement.

a) Subtract the product of 3 and -8 from 20.

b) Add the product of 4 and 5 to the product of -2 and -3.

c) Divide -62 by the sum of -11 and 9.

d) Multiply the sum of -3 and -5 by 3, then divide by -4 and subtract 13.

Extend

21. Copy each statement. Complete it by including operation symbols.

a) 2 ▪ 3 ▪ 4 ▪ 5 = -14

b) 3 ▪ [14 ▪ (-2)] ▪ 30 = 6

22. The mean of two integers is -17. The product of the two integers is 273. What are the two integers?

23. A multiple-choice test with 50 questions has five possible choices for each question. There are 4 marks for each correct answer, -1 mark for each incorrect answer, and 0 marks for each unanswered question.

a) What is the total score of a student with 35 correct answers, 10 incorrect answers, and 5 unanswered questions?

b) Express the student's score as a percent.

24. Here is one way of using four -2s and the order of operations to write an expression that equals 1.

$(-2) \div (-2) + (-2) - (-2)$

Use four -2s and the order of operations to write expressions that equal 2, 3, 4, 5, 6, and 8.

Sports Link

Scoring System in the Modern Pentathlon

Monica Pinette from Langley, British Columbia, is a successful Canadian athlete. She has won gold medals at both the Canadian and the Pan American championships in the modern pentathlon. This sport includes five events: shooting, fencing, swimming, show jumping, and running. The events are all held on the same day. The winner is decided using a points system that involves integer operations.

The running event is a 3000-m cross-country race. A male athlete scores 1000 points for finishing this event in 10-min. A female athlete scores 1000 points for finishing in 11 min 20 s. Each whole second below these times is worth 4 extra points. Each whole second above these times results in a 4-point deduction.

1. Show how the following points are calculated.

a) 920 points for a male with a time of 10 min 20 s

b) 1060 points for a female with a time of 11 min 5 s

2. Calculate the points earned by each of the following athletes.

a) a female with a time of 11 min 43 s

b) a male with a time of 9 min 51 s

3. Calculate the time taken by each of the following athletes.

a) a female who scores 1100 points

b) a male who scores 892 points

Did You Know?

Another sport that includes several events is the decathlon. Dave Steen from New Westminster, British Columbia, won a bronze medal in the decathlon at the summer Olympics in Seoul, South Korea. He competed in ten events over two days.

WWW Web Link

To learn more about the modern pentathlon, including the scoring of the other four events, go to www.mathlinks8.ca and follow the links.