Dividing Integers

Farmers around the world use fertilizers made from potash mined in Saskatchewan. The province produces over 40% of the world's supply of potash.

To reach the potash, miners are lowered down a vertical mineshaft in a cage. Typical mineshafts are 900 m to 1100 m deep. The cage descends at about 6 m/s. How could you use integer chips to determine the time it takes to descend 900 m? Describe any difficulty you see in using integer chips to determine the time.
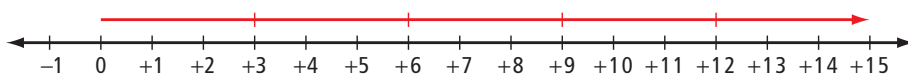
## Explore the Math

Materials
• red and blue integer chips

How can you divide two integers?

1. The diagram shows how you can model the division $(+15) \div (+3)$ using a number line.

a) How are the two integers in the division $(+15) \div (+3)$ shown in the diagram?

b) Model $(+15) \div (+3)$ using integer chips. What is the quotient?

c) How does the number line show the quotient?

d) Explain how the diagram can also model the division $(+15) \div (+5)$.

2. The diagram shows how you can model the division $(-15) \div (-3)$ using a number line.
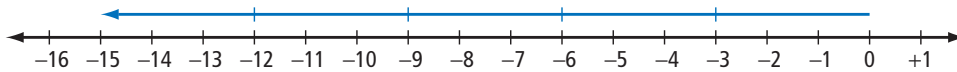
a) How are the two integers in the division $(-15) \div (-3)$ shown in the diagram?

b) Model $(-15) \div (-3)$ using integer chips. What is the quotient?

c) How does the number line show the quotient?

d) Explain how the diagram can also model the division $(-15) \div (+5)$.

3. a) Model the division $(-15) \div (+3)$ using a number line. Explain your reasoning.

b) Copy and complete the division statement $(-15) \div (+3) = \blacksquare$.

c) Explain how your diagram can also model the division $(-15) \div (-5)$.

4. Can you use the same methods as in #1 to #3 to model the division $(+15) \div (-3)$? Explain.

5. The first row of the table shows a multiplication statement and the two division statements related to it. Copy and complete the table.

Multiplication Statement	Related Division Statements	
$(+2) \times (+4) = +8$	$(+8) \div (+4) = +2$	$(+8) \div (+2) = +4$
$(+6) \times (+2) = +12$		
$(+3) \times (-5) = -15$		
$(-3) \times (+6) = -18$		
$(-5) \times (-4) = +20$		
$(-1) \times (-9) = +9$		

6. Copy each of the following statements. Use your results from the table to complete each statement using the word "positive" or the word "negative."

The quotient of two integers with the same sign is ▮.
The quotient of two integers with different signs is ▮.

Reflect on Your Findings

7. a) How can you use a number line to divide two integers? In your description, state any limitations of your method.

b) How can you use the signs of two integers to help determine their quotient?

Example 1: Divide Integers

Calculate.

a) $(+6) \div (+2)$ **b)** $(-12) \div (-6)$
c) $(-20) \div (+4)$ **d)** $(+42) \div (-14)$

Solution
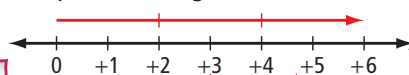
Divide the numerals and then apply a **sign rule**.

a) $6 \div 2 = 3$
The integers $+6$ and $+2$ have the same sign, so the quotient is positive.
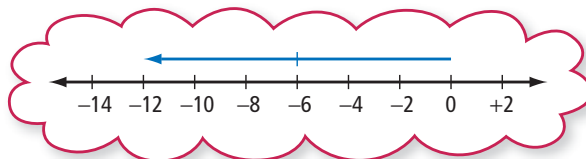$(+6) \div (+2) = +3$

> You can also determine the quotient using a number line.

b) $12 \div 6 = 2$
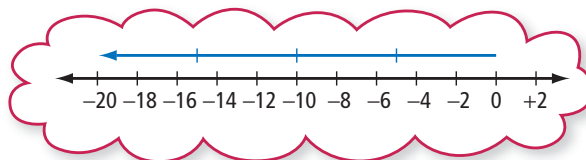The integers -12 and -6 have the same sign, so the quotient is positive.
$(-12) \div (-6) = +2$

c) $20 \div 4 = 5$
The integers -20 and $+4$ have different signs, so the quotient is negative.
$(-20) \div (+4) = -5$

d) $42 \div 14 = 3$
The integers $+42$ and -14 have different signs, so the quotient is negative.
$(+42) \div (-14) = -3$

C **42** ÷ **14** +∿− = -3.

Check:
$(-3) \times (-14) = +42$

> You can use multiplication to check your division.

Show You Know

Calculate.

a) $(+24) \div (+8)$ **b)** $(+30) \div (-10)$
c) $(-48) \div (-12)$ **d)** $(-66) \div (+11)$

Example 2: Apply Integer Division

Daria and four of her friends went out for lunch. They agreed to split the cost equally. The total bill came to $85, which Daria paid on her credit card. How much did each of her friends owe Daria?

Solution

Use the division of two integers to represent the situation.
Represent the total cost of $85 by the integer -85.
Represent the 5 people by the integer $+5$.
Each person's share can be represented by the expression $(-85) \div (+5)$.
$(-85) \div (+5) = -17$
Each of her friends owed Daria $17.

> $85 \div 5 = 17$. The integers -85 and $+5$ have different signs, so the quotient is negative.

Check.
Use multiplication to check the division.
$(-17) \times (+5) = -85$

Show You Know

Pierre paid $42 to admit himself and two of his friends into a science museum. What was the cost of each admission?

Key Ideas

- You can model some integer divisions on a number line.
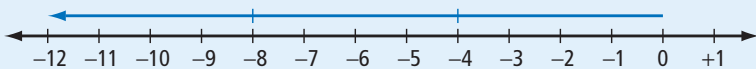
$(-12) \div (-4) = +3$ $(-12) \div (+3) = -4$

- You can divide two integers by dividing the numerals and applying the sign rules.
  - The quotient of two integers with the same sign is positive.
    $(+6) \div (+2) = +3$ $(-6) \div (-2) = +3$
  - The quotient of two integers with different signs is negative.
    $(+6) \div (-2) = -3$ $(-6) \div (+2) = -3$

Communicate the Ideas

1. To model the division $(+15) \div (+3)$ on a number line, you first draw an arrow that represents $+15$. You then have two choices:
   - You can cut the arrow into parts that each represent $+3$ and count how many parts there are.
   - You can cut the arrow into three equal parts and determine the value that each part represents.

   Which choice do you prefer? Explain.

2. Aziza used a number line to model the division $(-12) \div (-2)$. Yuri used a number line to model the division $(-12) \div (+6)$. They drew the same diagram. What was the diagram?

3. Michel said, "When I divide $+6$ by $+3$, $+2$, or $+1$, the quotient is less than or equal to $+6$. If I divide -6 by $+3$, $+2$, or $+1$, I think the quotient should be less than or equal to -6." Do you agree with him? Explain.

4. Without doing any calculations, Stefani said that the quotients $(-252) \div (-18)$ and $(+252) \div (+18)$ must be the same. How did she know?
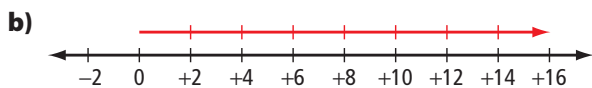
Check Your Understanding

Practise

For help with #5 to #10, refer to Example 1 on page 308.
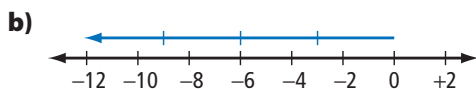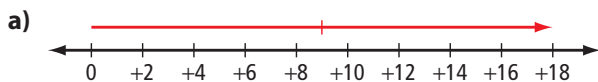
5. Write two division statements that each diagram could represent.

a)

b)

6. Write two division statements that each diagram could represent.

a)

b)

7. Determine each quotient using a number line.
  a) $(+12) \div (+6)$ **b)** $(-20) \div (-4)$
  c) $(-8) \div (+4)$ **d)** $(-10) \div (-5)$

8. Determine each quotient using a number line.
  a) $(-14) \div (-7)$ **b)** $(+16) \div (+4)$
  c) $(-22) \div (+2)$ **d)** $(-15) \div (-5)$

9. Calculate and check.
  a) $(+20) \div (+5)$ **b)** $(+36) \div (-6)$
  c) $(-57) \div (+19)$ **d)** $(-84) \div (-42)$

10. Calculate.
  a) $(-26) \div (-26)$ **b)** $(+95) \div (-5)$
  c) $0 \div (-33)$ **d)** $(-68) \div (+17)$

Apply

For help with #11 to #15, refer to Example 2 on page 309. Use the division of two integers to represent each situation and solve the problem.

11. Raoul borrowed $15 per month from his mother to pay for the art supplies he needed for an evening class. At the end of the course, he owed his mother $60. How long was the course?

12. a) A submarine took 16 min to dive 96 m from the surface. How far did it dive per minute?

  b) The submarine took 12 min to climb back to the surface. How far did it climb per minute?

13. A scuba diver was collecting water samples from a lake. He collected samples at 5-m intervals starting at 5 m below the surface. He collected the final sample at a depth of 35 m. How many samples did he collect?

14. Mina was drilling down through a 21-cm thick concrete floor to install a new plumbing pipe. She drilled for 5 min, took a break, and then finished drilling in another 2 min. At what rate did the drill cut through the floor, in centimetres per minute? What assumptions did you make?

15. A school spent $384 to buy a set of 32 calculators. What was the cost of each calculator?

16. Without evaluating the quotients, identify the quotient with the least value. Explain your reasoning.
$(+2408) \div (+43)$
$(-2408) \div (-43)$
$(+2408) \div (-43)$

17. If 28 times an integer is -448, what is the integer?

18. Copy and complete each statement.
   a) $(+72) \div (\blacksquare) = +9$
   b) $(\blacksquare) \div (+12) = -10$
   c) $(\blacksquare) \div (-13) = -11$
   d) $(-84) \div (\blacksquare) = +6$

19. Write a word problem that you can solve using the expression $(-80) \div (+16)$.

20. Create your own word problem that involves integer division. Make sure that you can solve your problem and that the calculation results in an integer. Give your problem to a classmate to solve.

Extend

21. Describe each pattern. Then write the next three terms in each pattern.
   a) $+125\,000, +25\,000, +5000, +1000, \dots$
   b) $-512, +256, -128, +64, \dots$
   c) $-1\,000\,000, -100\,000, -10\,000, -1000, \dots$
   d) $+1458, -486, +162, -54, \dots$

22. The sum of two integers is $+20$. Dividing the larger integer by the smaller integer gives a quotient of -3. What are the two integers?

MATH LINK

The temperature of still, dry air decreases by about 6 °C for each kilometre increase in altitude. On a still, dry day, the temperature in Yellowknife, Northwest Territories, was −11 °C. The air temperature outside a plane flying above Yellowknife was −53 °C.
a) Approximately how much lower was the temperature outside the aircraft than the temperature in Yellowknife?
b) How high was the aircraft above Yellowknife?