# 8.2 Multiplying Integers

## Focus on...

After this lesson, you will be able to...

- ❑ determine integer products using a number line
- ❑ apply a sign rule when multiplying integers

Birds called sooty shearwaters have the longest known migration of any animal. Huge flocks of these birds leave their breeding grounds in New Zealand as winter approaches. They fly across and around the Pacific Ocean to take advantage of summer in the Northern Hemisphere. Some of them visit the coastal waters of British Columbia. The birds head south again as winter approaches in the North. Scientists have measured the birds' annual migration at about 70 000 km.

Sooty shearwaters feed by diving into the ocean to catch fish, squid, and krill. The birds dive to an average depth of 14 m. Their deepest dives are about five times as deep as that. How could you use integers to determine the depth of their deepest dives?

**Did You Know?**

For many years, scientists thought that the arctic tern was the distance champion of bird migration. This bird breeds in the Arctic and migrates to Antarctica and back each year. The distance that it covers is at least 35 000 km.

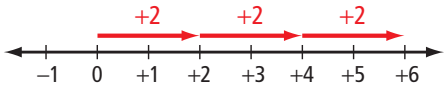
**Materials**

- red and blue integer chips

## Explore the Math

### How can you multiply two integers?

**1.** The diagram shows how you can model the multiplication $(+3) \times (+2)$ using a number line.




**a)** How are the two integers in the multiplication $(+3) \times (+2)$ shown in the diagram?

**b)** Model $(+3) \times (+2)$ using integer chips. What is the product?

**c)** How does the number line show the product?

**2. a)** Model the multiplication $(+4) \times (-3)$ using a number line. Explain your reasoning.

**b)** What is the product? Explain how you know.

**3.** Can you use the same method as in #1 or #2 to model $(-3) \times (+2)$ or $(-4) \times (-3)$ using a number line? Explain.

**4. a)** Copy the table. Use a suitable model to help you complete each multiplication statement.

| | |
|---|---|
| $(+6) \times (+2) =$ ■ | $(+2) \times (+6) =$ ■ |
| $(+4) \times (-5) =$ ■ | $(-5) \times (+4) =$ ■ |
| $(-4) \times (-3) =$ ■ | $(-3) \times (-4) =$ ■ |

**b)** Compare the two multiplication statements on each row of the completed table. What can you conclude about the order in which you can multiply two integers? Test your conclusion on some other integer multiplications.

**c)** From your answer to part b), describe a way to determine $(-3) \times (+2)$ using a number line.

**5. a)** Copy each of the following statements. Use your results from the table in #4 to complete each statement using the word "positive" or the word "negative."

The product of two integers with the same sign is ■.
The product of two integers with different signs is ■.

**b)** Test your statements from part a) on some other integer multiplications.

## Reflect on Your Findings

**6. a)** How can you use a number line to multiply two integers? In your description, state any limitations of your method.

**b)** How can you use the signs of two integers to help determine their product?

## Example 1: Multiply Integers

Calculate.

**a)** $(+3) \times (+4)$ **b)** $(+2) \times (-9)$ **c)** $(-5) \times (+6)$ **d)** $(-6) \times (-4)$

**sign rule**

(for multiplication)

- the product of two integers with the same sign is positive
- the product of two integers with different signs is negative

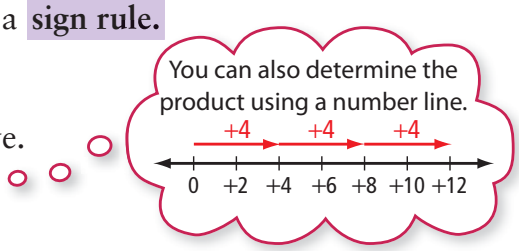
### *Solution*

Multiply the numerals and then apply a **sign rule.**

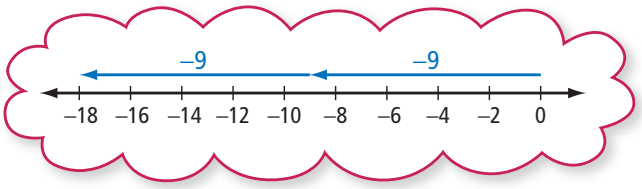
**a)** $3 \times 4 = 12$

The integers $+3$ and $+4$ have the same sign, so the product is positive.

$(+3) \times (+4) = +12$



**b)** $2 \times 9 = 18$
The integers $+2$ and $-9$ have different signs, so the product is negative.
$(+2) \times (-9) = -18$




**c)** $5 \times 6 = 30$
The integers $-5$ and $+6$ have different signs, so the product is negative.
$(-5) \times (+6) = -30$

**d)** $6 \times 4 = 24$
The integers $-6$ and $-4$ have the same sign, so the product is positive.
$(-6) \times (-4) = +24$

### Show You Know

Calculate.

**a)** $(+4) \times (+7)$ **b)** $(+3) \times (-10)$ **c)** $(-8) \times (-2)$ **d)** $(-4) \times (+9)$

## Example 2: Apply Integer Multiplication

Tina supports her favourite charity with an automatic deduction of $35/month from her bank account. Estimate and then calculate the total of her deductions in a year?

### *Solution*

Use the multiplication of two integers to represent the situation.
Represent the $35 deduction each month by the integer $-35$. Represent the number of monthly deductions in a year by the integer $+12$.

You could also write $(-35) \times (+12)$, because the order of multiplication does not matter.

The total of the deductions can be represented by the product $(+12) \times (-35)$.

Estimate the product.
$12 \times 35 \approx 10 \times 40$
so $(+12) \times (-35) \approx (+10) \times (-40)$
$\approx -400$


The integers $+12$ and $-35$ have different signs, so the product is negative.

Multiply $(+12) \times (-35)$ using the sign rules.
$(+12) \times (-35) = -420$
The total of her deductions in a year is $420.

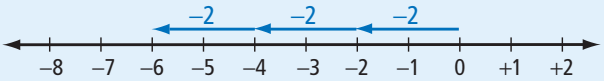
The negative sign shows that Tina's account has decreased by $420.

### Show You Know

Duane instructs his bank to deduct $65 per month from his bank account and transfer the money into an investment account. What is the total of his deductions in 18 months?

## Key Ideas

- You can model the multiplication of a positive integer by an integer on a number line.




$(+3) \times (-2) = -6$

- You can multiply two integers by multiplying the numerals and applying the sign rules:
  - The product of two integers with the same sign is positive.
    $(+2) \times (+5) = +10$
    $(-2) \times (-5) = +10$
  - The product of two integers with different signs is negative.
    $(+2) \times (-5) = -10$
    $(-2) \times (+5) = -10$
- Multiplying two integers in either order gives the same result.
  $(-5) \times (+3) = -15$
  $(+3) \times (-5) = -15$

## Communicate the Ideas

**1.** Darcy modelled $(+7) \times (+3)$ on a number line by drawing seven arrows. Ishnan modelled $(+7) \times (+3)$ on a number line by drawing only three arrows. Explain Ishnan's thinking.

**2.** Justin said, "When I multiply $+5$ by a negative integer, the product is less than $+5$. If I multiply $-5$ by a negative integer, I think the product should be less than $-5$." Do you agree with him? Explain.

**3.** Without doing any calculations, Wei said that $-19$ and $+27$ have the same product as $+19$ and $-27$. How did she know?

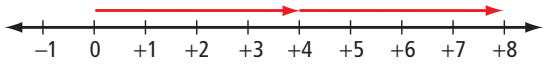
# Check Your Understanding

## Practise

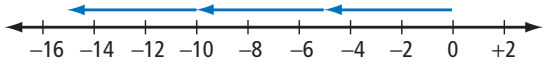
*For help with #4 to #9, refer to Example 1 on pages 294–295.*

**4.** What multiplication statement does each diagram represent?

**a)**

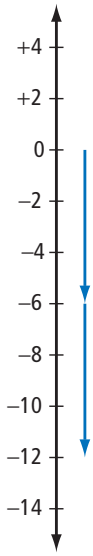



**b)**




**5.** What multiplication statement does each diagram represent?

**a)**




**b)**


**6.** Determine each product using a number line.

**a)** $(+5) \times (+5)$ **b)** $(+3) \times (-6)$

**7.** Determine each product.

**a)** $(+4) \times (-7)$ **b)** $(+2) \times (+9)$

**8.** Determine each product using the sign rules.

**a)** $(+10) \times (+4)$ **b)** $(+6) \times (-5)$

**c)** $(-7) \times (+5)$ **d)** $(-8) \times (-4)$

**9.** Determine each product.

**a)** $(-6) \times (-6)$ **b)** $(+9) \times (+6)$

**c)** $(-12) \times (+2)$ **d)** $(+11) \times 0$

*For help with #10 to #15, refer to Example 2 on page 295.*

**10.** Estimate and then calculate.

**a)** $(+17) \times (-24)$

**b)** $(+37) \times (+22)$

**c)** $(-72) \times (+15)$

**d)** $(-28) \times (-47)$

**11.** Estimate and then calculate.

**a)** $(-18) \times (-14)$

**b)** $(-51) \times (+26)$

**c)** $(+99) \times (+12)$

**d)** $(+55) \times (+55)$

## Apply

*For #12 to #15, use the multiplication of two integers to represent each situation.*

**12.** A telephone company offers its customers a $15 discount per month if they also sign up for Internet service. How much is the annual discount?

**13.** A hot-air balloon is descending at 60 m/min. How far does it descend in 25 min?

**14.** Ana owns 75 shares of the Leafy Greens Company. One week, the value of each share dropped by 60¢. The next week, the value of each share grew by 85¢. What was the total change in the value of Ana's shares

**a)** in the first week?

**b)** in the second week?

**c)** over the two-week period?

**15.** To prepare for the weightlessness of space, astronauts train using steep dives on an aircraft. In one dive, the aircraft can descend at 120 m/s for 20 s. How far does the aircraft descend?

**16.** In the following list of integers, identify the two integers that have the greatest product.

$$+21, -18, +12, +14, -23, -15, +19, -13$$

**17.** Without evaluating the products, identify the least product. Explain your reasoning.

$(+99) \times (+82)$
$(-99) \times (-82)$
$(+99) \times (-82)$

**18.** Suppose a friend knows how to multiply positive integers but has never multiplied negative integers.

**a)** How could you use the following pattern to show your friend how to calculate $(+5) \times (-3)$?

$(+5) \times (+3) = +15$
$(+5) \times (+2) = +10$
$(+5) \times (+1) = +5$
$(+5) \times 0 = 0$
$(+5) \times (-1) =$ ■
$(+5) \times (-2) =$ ■
$(+5) \times (-3) =$ ■

**b)** Make up a pattern to show your friend how to calculate $(+6) \times (-2)$.

**19. a)** Can $+4$ be written as the product of two equal integers? Explain.

**b)** Can $-4$ be written as the product of two equal integers? Explain.

**20.** Copy and complete each multiplication statement.

**a)** $(+6) \times$ ■ $= +18$

**b)** ■ $\times (-2) = -10$

**c)** ■ $\times (+3) = -12$

**d)** $(-4) \times$ ■ $= +16$

**21.** Complete each statement in as many ways as possible using integers.

**a)** ■ $\times$ ■ $= +10$

**b)** ■ $\times$ ■ $= -16$

**c)** ■ $\times$ ■ $= -24$

**22.** The sum of two integers is $-5$. The product of the same two integers is $-36$. What are the two integers?

**23.** Write a word problem that you can solve using the expression $(+5) \times (-6)$.

**24.** Create your own word problem that involves integer multiplication. Make sure that you can solve your problem. Give your problem to a classmate to solve.

## Extend

**25.** Describe each pattern. Then write the next three terms in each pattern.

**a)** $+1, +3, +9, +27, \ldots$

**b)** $-1, +2, -4, +8, \ldots$

**c)** $-2, -4, -8, -16, \ldots$

**d)** $+2, -8, +32, -128, \ldots$

**26.** For each statement, describe a situation in which the statement is true.

**a)** The product of two integers equals one of the integers.

**b)** The product of two integers equals the opposite of one of the integers.

**c)** The product of two integers is less than both integers.

**d)** The product of two integers is greater than both integers.

**Literacy Link**

Two integers with the same numeral but different signs are called opposite integers. Examples are +5 and −5.

**27. a)** Identify three consecutive integers whose sum and product both equal zero.

**b)** Repeat part a) for five consecutive integers.

**c)** Can you repeat part a) for two consecutive integers or for four consecutive integers? Explain.

**28.** In a magic multiplication square, the numbers in each row, column, and diagonal have the same product. This is called the magic product.

**a)** What is the magic product of this square?

| | | |
|---|---|---|
| +12 | −1 | +18 |
| −9 | −6 | −4 |
| +2 | −36 | +3 |

**b)** Multiply each number in the square from part a) by −2. Is the result a magic multiplication square? If so, what is the magic product?

**c)** Add −5 to each number in the square from part a). Is the result a magic multiplication square? If so, what is the magic product?

**29.** Write a sign rule for the product of each of the following.

**a)** an even number of positive integers

**b)** an odd number of positive integers

**c)** an even number of negative integers

**d)** an odd number of negative integers

## MATH LINK

The temperature of still, dry air decreases by about 6 °C for each kilometre increase in altitude. A weather balloon was launched from The Pas, Manitoba, on a still, dry day.

**a)** If the temperature on the ground was +4 °C, what was the approximate temperature 11 km above the ground?

**b)** If the balloon then descended to 5 km above ground, about how much did the temperature change during the descent?