Exploring Integer Multiplication

Did You Know?

The Columbia Icefield is a major source of fresh water. Melt water from the icefield feeds rivers that flow to the Arctic Ocean, the Pacific Ocean, and Hudson's Bay.

The Columbia Icefield is the largest mass of ice in North America below the Arctic Circle. The icefield lies across the Alberta–British Columbia border in the Rockies. Six large glaciers flow from the icefield. One of them, the Athabasca Glacier, is a popular tourist destination in Jasper National Park.

The Athabasca Glacier has been melting for over a century. The front edge or "snout" of the glacier has been receding at an average of approximately 12 m per year. At that rate, how far would it recede in four years?

Explore the Math

Materials
• red and blue integer chips

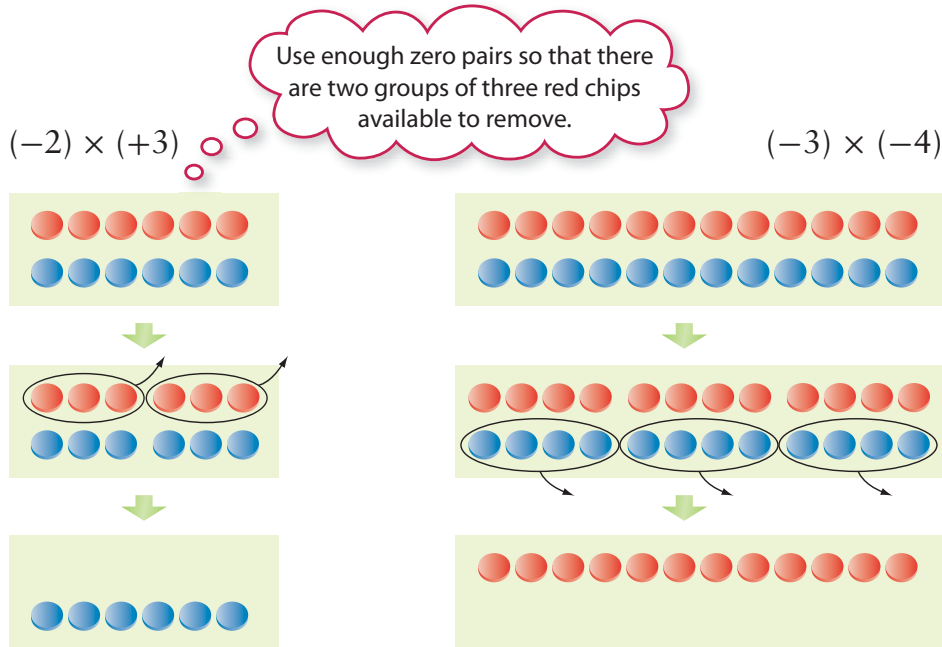**Literacy Link**

Representing Integers

Integer chips are coloured disks that represent integers. In this book, one red chip represents +1, and one blue chip represents −1.

+1   −1

How can you use integer chips to multiply two integers?

1. Multiplication can be expressed as a repeated addition.

$(+3) \times (+2) = (+2) + (+2) + (+2)$

   a) Use red integer chips to model the addition $(+2) + (+2) + (+2)$.

   b) Copy and complete the multiplication statement $(+3) \times (+2) = \blacksquare$.

2. **a)** Express $(+4) \times (+3)$ as a repeated addition.

   b) Use red integer chips to model the addition.

   c) Copy and complete the multiplication statement $(+4) \times (+3) = \blacksquare$.

3. **a)** Express $(+3) \times (−5)$ as a repeated addition.

   b) Use blue integer chips to model the addition.

   c) Copy and complete the multiplication statement $(+3) \times (−5) = \blacksquare$.

4. Use integer chips to model each multiplication. Copy and complete each multiplication statement.

a) $(+3) \times (+5) = \blacksquare$ **b)** $(+2) \times (+3) = \blacksquare$

c) $(+4) \times (-3) = \blacksquare$ **d)** $(+3) \times (-4) = \blacksquare$

5. Can you use the same method as in #1 to #4 to complete the multiplication $(-2) \times (+3)$ or the multiplication $(-3) \times (-4)$? Explain.

6. a) The diagrams show how to model the multiplications from #5 by using zero pairs. Describe each model.

Use enough zero pairs so that there are two groups of three red chips available to remove.

$(-2) \times (+3)$

$(-3) \times (-4)$

b) Copy and complete each multiplication statement.

$(-2) \times (+3) = \blacksquare$

$(-3) \times (-4) = \blacksquare$

7. Use integer chips to model each multiplication. Copy and complete each multiplication statement.

a) $(-4) \times (-3) = \blacksquare$ **b)** $(-5) \times (+2) = \blacksquare$

c) $(-2) \times (+4) = \blacksquare$ **d)** $(-1) \times (-4) = \blacksquare$

Reflect on Your Findings

8. How can you use integer chips to multiply two integers? In your description, state when you use zero pairs. How do you determine the number of zero pairs to use?
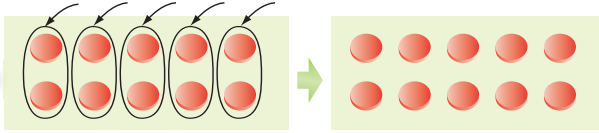
Example 1: Multiply Using Integer Chips

Determine each product using integer chips. Copy and complete each multiplication statement.

a) $(+5) \times (+2)$ **b)** $(+6) \times (-2)$ **c)** $(-3) \times (+2)$ **d)** $(-2) \times (-4)$
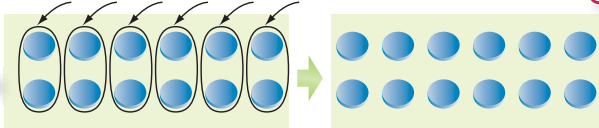
Solution

a)

$(+5) \times (+2) = (+2) + (+2) + (+2) + (+2) + (+2)$
Insert 5 groups with 2 red chips in each group.

There are 10 red chips.

The product is $+10$.
The multiplication statement is $(+5) \times (+2) = +10$.

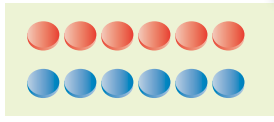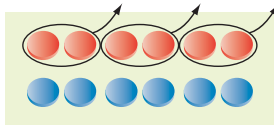**b)**

$(+6) \times (-2) = (-2) + (-2) + (-2) + (-2) + (-2) + (-2)$
Insert 6 groups with 2 blue chips in each group.
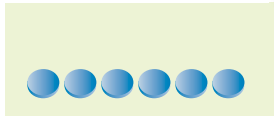
There are 12 blue chips.

The product is -12.
The multiplication statement is $(+6) \times (-2) = -12$.

c) The negative sign in -3 shows that you need to remove 3 groups. Each group will contain 2 red chips. So, you need to remove a total of 6 red chips.

Start with 6 zero pairs, so that there are enough red chips to remove.

Now remove the red chips.

There are 6 blue chips.

The product is -6.
The multiplication statement is $(-3) \times (+2) = -6$.

d) The negative sign in −2 shows that you need to remove 2 groups. Each group will contain 4 blue chips. So, you need to remove a total of 8 blue chips.

Start with 8 zero pairs, so that there are enough blue chips to remove.

Now remove the blue chips.

There are 8 red chips.
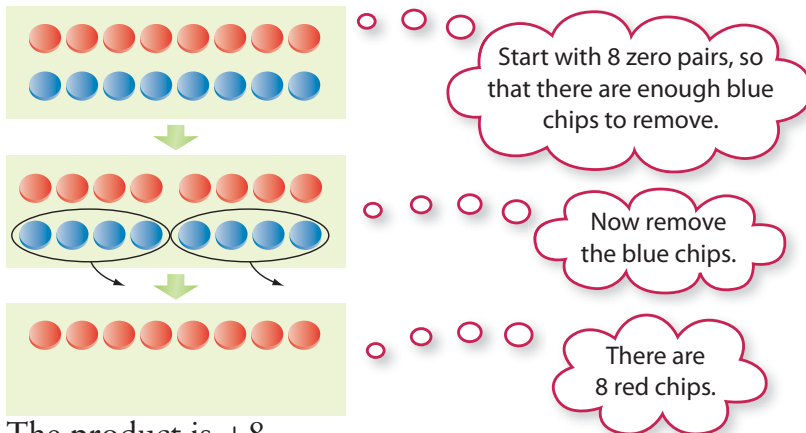
The product is +8.
The multiplication statement is $(-2) \times (-4) = +8$.

Show You Know

Determine each product using integer chips. Use diagrams to show your thinking.

a) $(+4) \times (+2)$ **b)** $(+5) \times (-2)$ **c)** $(-4) \times (+2)$ **d)** $(-6) \times (-1)$

Example 2: Apply Integer Multiplication

For 5 h, the temperature in Flin Flon fell by 3 °C each hour. What was the total change in temperature?

Solution

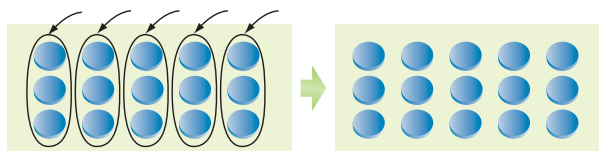Use the multiplication of two integers to represent the situation.
Represent the time of 5 h by the integer +5.
Represent the 3 °C decrease in each hour by the integer −3.
The total change in temperature can be represented
by the expression $(+5) \times (-3)$.

Multiply $(+5) \times (-3)$ using integer chips.

The number of hours times the change per hour gives the total change.

The product is −15.
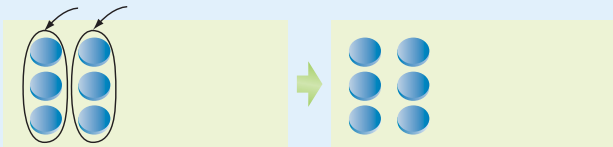The total change in temperature was a decrease of 15 °C.

Show You Know

For 4 h, the temperature in Victoria fell by 2 °C each hour. What was the total change in temperature?

Key Ideas

- To model the multiplication of an integer by a positive integer, you can insert integer chips of the appropriate colour.

$(+2) \times (-3) = -6$

- To model the multiplication of an integer by a negative integer, you can remove integer chips of the appropriate colour from zero pairs.
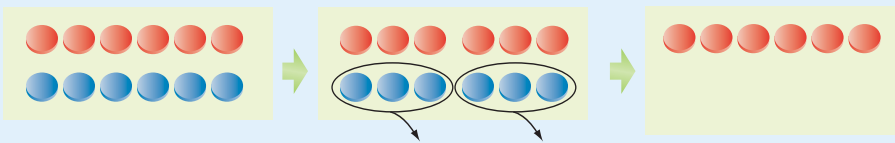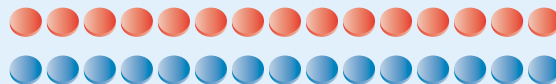
$(-2) \times (-3) = +6$

Communicate the Ideas

1. David said that he could model the multiplication $(+3) \times (-7)$ using 3 red chips and 7 blue chips.
   a) Do you agree with David? Explain.
   b) What chips would you use to model $(+3) \times (-7)$? Explain.

2. To model $(-3) \times (-5)$, Raini places 15 zero pairs on her desk.
   a) Why did she use 15 zero pairs?
   b) What should she do next?

3. a) Paolo models $(-2) \times (+3)$ as shown in the diagram. He determines the correct product, -6. Explain why his method works.
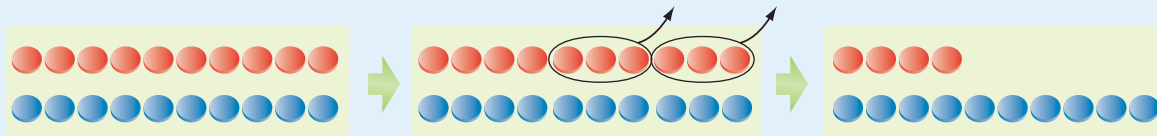
   b) Could Paolo model the product if he started with 4 zero pairs? Explain.

4. Jasmine said that she did not need any integer chips to multiply $0 \times (+2)$ or to multiply $(-3) \times 0$. Explain her thinking.

Practise

For help with #5 to #14, refer to Example 1 on pages 288–289.

5. Write each repeated addition as a multiplication.

a) $(+1) + (+1) + (+1) + (+1) + (+1)$

b) $(-6) + (-6)$

6. Write each expression as a multiplication.

a) $(+7) + (+7) + (+7)$

b) $(-4) + (-4) + (-4) + (-4)$

7. Write each multiplication as a repeated addition.

a) $(+3) \times (+8)$ **b)** $(+5) \times (-6)$

8. Write each expression as a repeated addition.

a) $(+7) \times (+2)$ **b)** $(+4) \times (-9)$

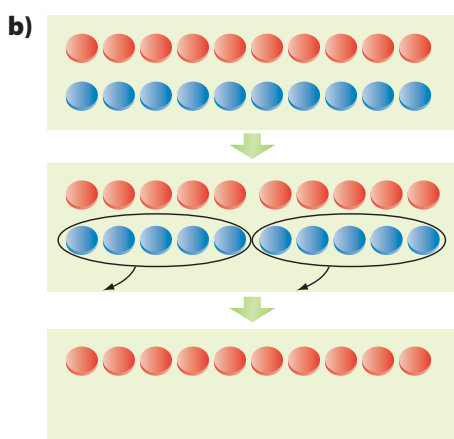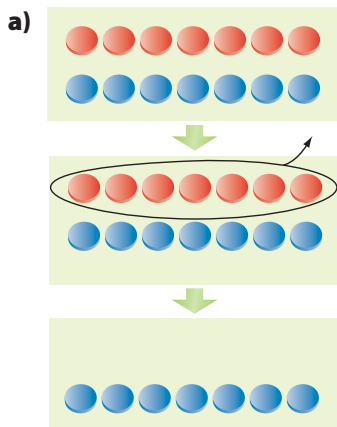**9.** What multiplication statement does each set of diagrams represent?
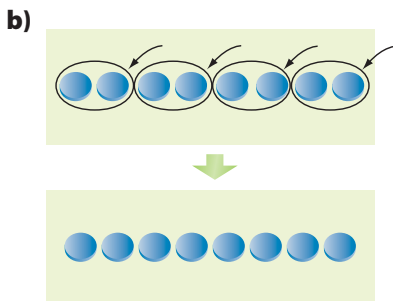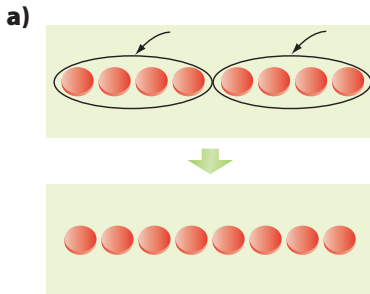
a)

b)

10. What multiplication statement does each set of diagrams represent?

a)

b)

11. What multiplication statement does each set of diagrams represent?

a)

b)
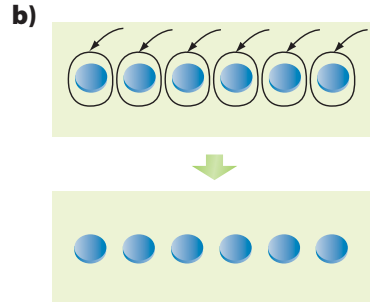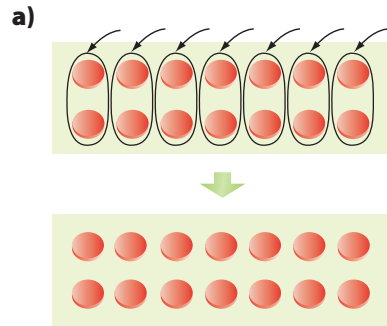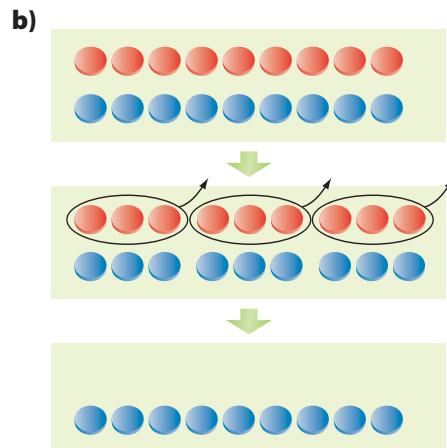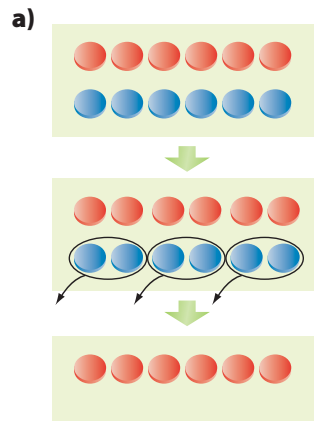
12. What multiplication statement does each set of diagrams represent?

a)

b)

13. Copy and complete each multiplication statement.

a) $(+4) \times (+6)$
b) $(+7) \times (-2)$
c) $(-1) \times (+5)$
d) $(-8) \times (-2)$

Apply

For help with #14 to #17, refer to Example 2 on page 289.

14. Use the multiplication of two integers to represent each situation. Then determine the product and explain its meaning.

a) The temperature increased for 6 h at 2 °C per hour.

b) Ayesha repaid some money she owed in 4 instalments of $8 each.

15. An aircraft descends at 3 m/s for 12 s. Use the multiplication of two integers to represent the situation. How far does the aircraft descend?

16. A building has 10 storeys above ground and 3 storeys below ground. Each storey has a height of 4 m.

a) What is the total height of the building above ground?

b) What is the total depth of the building below ground?

17. An oil rig is drilling a well at 2 m/min. How deep is the well after the first 8 min?

18. Does doubling an integer always result in an integer of greater value? Explain.

Extend

19. In a magic square, the numbers in each row, column, and diagonal have the same sum. This is called the magic sum.

a) What is the magic sum for this magic square?

+2	+3	−2
−3	+1	+5
+4	−1	0

b) Multiply each integer in the square by −2. Is the result another magic square? If so, what is the new magic sum?

c) Create a magic square with a magic sum of −12.

20. Arrange the following numbers of +1s and −1s in the small squares on a three-by-three grid so that each row, column, and diagonal has a product of −1.

a) six +1s, three −1s

b) four +1s, five −1s