Applying Fraction Operations

Gold has been valued since ancient times because of its beauty and its short supply. Canada is one of the world's leading gold producers.

About $\frac{17}{20}$ of the world's gold production is used to make jewellery.

About $\frac{1}{40}$ of the world's gold production is used to make coins. What operation would you use to determine how many times as much gold is used to make jewellery as is used to make coins?

Explore the Math

How can you decide which operations to use when solving problems involving fractions?

Many objects that appear to be made of pure gold are actually made from mixtures of gold and cheaper metals. The purity of the gold is measured using a unit called the karat. This unit represents the fraction $\frac{1}{24}$. The table shows the fraction of gold and the fraction of other metals in gold objects with two different purities.

Purity of Gold (karats)	Fraction of Gold	Fraction of Other Metals
20	$\frac{5}{6}$	$\frac{1}{6}$
14	$\frac{7}{12}$	$\frac{5}{12}$

1. How would you calculate the fraction of gold from the purity of gold? Explain.

2. How would you calculate the fraction of other metals from the fraction of gold? Explain.

3. For a gold object with a purity of 18 karats, what is

    a) the fraction of gold, in lowest terms?

    b) the fraction of other metals, in lowest terms?

4. Use the meaning of a karat to explain why pure gold is described as 24-karat gold.

5. How would you calculate the purity of gold from the fraction of gold? Explain.

6. What is the purity of gold, in karats, if the fraction of gold is

    a) $\dfrac{1}{2}$?   **b)** $\dfrac{5}{12}$?

Reflect on Your Findings

7. How did you decide which operations to use in #1, #2, and #5? Discuss your ideas with your classmates.

Example 1: Use the Order of Operations

Calculate.

a) $2 \div \dfrac{1}{4} + 3 \times \dfrac{1}{2}$ **b)** $\dfrac{1}{3} \times (9 - 2) - \dfrac{5}{6}$ **c)** $2\dfrac{1}{4} \div \left(1\dfrac{3}{4} + 1\dfrac{1}{4}\right)$

Solution

a) $2 \div \dfrac{1}{4} + 3 \times \dfrac{1}{2}$ Divide.

$= 8 + 3 \times \dfrac{1}{2}$ Multiply.

$= 8 + \dfrac{3}{2}$ Add.

$= \dfrac{16}{2} + \dfrac{3}{2}$

$= \dfrac{19}{2}$ or $9\dfrac{1}{2}$

b) $\dfrac{1}{3} \times (9 - 2) - \dfrac{5}{6}$ Brackets.

$= \dfrac{1}{3} \times 7 - \dfrac{5}{6}$ Multiply.

$= \dfrac{7}{3} - \dfrac{5}{6}$ Subtract.

$= \dfrac{14}{6} - \dfrac{5}{6}$

$= \dfrac{9}{6}$

$= \dfrac{3}{2}$ or $1\dfrac{1}{2}$

c) $2\dfrac{1}{4} \div \left(1\dfrac{3}{4} + 1\dfrac{1}{4}\right)$ Brackets.

$= 2\dfrac{1}{4} \div 3$ Divide.

$= \dfrac{9}{4} \times \dfrac{1}{3}$

$= \dfrac{9}{12}$

$= \dfrac{3}{4}$

Literacy Link

The *order* of *operations* for fractions is the same as for whole numbers and decimals.

- Brackets first.
- Multiply and divide in order from left to right.
- Add and subtract in order from left to right.

Literacy Link

To earn time-and-a-half means to be paid for $1\dfrac{1}{2}$ h for each hour of work done.

Example 2: Apply Fraction Operations

Bev earns \$25/h as a machine operator in a sawmill. For time worked above 40 h in a week, she earns time-and-a-half. How much does Bev earn for working 46 h in a week?

Solution

Method 1: Calculate in Stages
Bev's regular rate of pay is \$25/h. In 46 h, Bev works 40 h at her regular rate of pay and 6 h at time-and-a-half.

Amount earned at regular rate:
$40 \times 25 = 1000$

Bev works 6 h at time-and-a-half. Multiply to determine the number of hours Bev is paid for.
$6 \times 1\dfrac{1}{2} = 9$
Amount earned at time-and-a-half:
$9 \times 25 = 225$

Total earnings $= 1000 + 225$
$\qquad\qquad\quad = 1225$
Bev earns \$1225 for working 46 h in a week.

Method 2: Evaluate One Expression
Bev's regular rate of pay is \$25/h. In 46 h, Bev works 40 h at her regular rate of pay and 6 h at time-and-a-half.

For 6 h at time-and-a-half, Bev is paid for $1\dfrac{1}{2} \times 6$ h.

An expression that represents her total earnings is:
$$25 \times \left(40 + 1\dfrac{1}{2} \times 6\right)$$

Evaluate the expression using the order of operations.

$\quad 25 \times \left(40 + 1\dfrac{1}{2} \times 6\right)$ Brackets.
$= 25 \times 49$ Multiply.
$= 1225$
Bev earns \$1225 for working 46 h in a week.

Ron earns $15/h as a security guard. For time worked above 35 h in a week, he earns time-and-a-third. How much does Ron earn for working 41 h in a week?

Key Ideas

- You need to decide which operation(s) to perform on fractions to solve problems.
- Some fraction problems can involve the order of operations.
- The order of operations for fractions is the same as for whole numbers and decimals.
  - Brackets first.
  - Multiply and divide in order from left to right.
  - Add and subtract in order from left to right.

Communicate the Ideas

1. Ranjeet is entering a competition to win some gold coins. She must answer the following skill-testing question.

   What is the value of $10 - 2 \times \dfrac{1}{2}$?

   She is unsure if the correct answer is 4 or 9.

   a) How could Ranjeet determine a possible answer of 4?

   b) How could Ranjeet determine a possible answer of 9?

   c) What is the correct answer? Explain.

2. Dave and Manuel were comparing their solutions to the following problem.

   Three quarters of a number is 6. What is the number?

   Dave evaluated $\dfrac{3}{4} \times 6$ to get an answer of $4\dfrac{1}{2}$.

   Manuel evaluated $6 \div \dfrac{3}{4}$ to get an answer of 8.

   Which answer is correct? Explain.

3. Mia evaluated the expression $\left(\dfrac{1}{2} + \dfrac{1}{4}\right) \times \dfrac{5}{3}$ to equal $\dfrac{11}{12}$.

   a) What mistake did she make?

   b) What is the correct value?

Check Your Understanding

Practise

4. Calculate.

a) $\dfrac{3}{4} - \dfrac{1}{2} \times \dfrac{2}{3}$

b) $2\dfrac{1}{5} \div \left(\dfrac{4}{5} - \dfrac{1}{4}\right)$

c) $3\dfrac{1}{2} + 2\dfrac{1}{2} \times \left(1\dfrac{1}{4} - \dfrac{3}{4}\right)$

5. Calculate.

a) $\left(\dfrac{5}{6} + \dfrac{2}{3}\right) \times \dfrac{3}{7}$

b) $\dfrac{1}{2} + \dfrac{3}{5} \div \dfrac{3}{4} \div \dfrac{2}{5}$

c) $1\dfrac{2}{5} \times 2\dfrac{1}{2} \div \left(1\dfrac{1}{8} - \dfrac{2}{3}\right)$

Apply

6. Leo earns $16/h as a gardener in a city park. For time worked above 35 h in a week, he earns time-and-a-half. How much does he earn for each of the following numbers of hours worked in a week?

a) 36 h   b) 39 h   c) 42 h   d) $37\dfrac{1}{2}$ h

7. Two thirds of the land on a farm is used for grazing beef cattle. The rest of the land is used to grow crops. Half of the land for crops is used to grow corn. What fraction of the land on the farm is used to grow corn?

8. Melissa and Shinzo found $\dfrac{1}{2}$ a pitcher of iced tea in the fridge. They equally shared $\dfrac{3}{4}$ of the iced tea.

a) What fraction of a pitcher of iced tea did each of them drink?

b) What fraction of a pitcher of iced tea was left over?

9. Five sevenths of the 28 students in a grade 8 class visited a science museum on a field trip. How many students did not go on the trip? Solve the problem in two different ways.

10. Brass is an alloy that contains the metals copper and zinc. Copper typically accounts for $\dfrac{3}{5}$ of the mass of a piece of brass.

a) What is the mass of copper in 175 g of brass?

b) What mass of brass contains 90 g of copper?

c) What mass of brass contains 50 g of zinc?

11. The advertising space in a hockey team's yearbook is sold in fractions of a page. The advertising space sold in one edition of the yearbook is shown in the table.

Size of Advertisement	Price	Number Sold
$\dfrac{1}{2}$ page	$110	3
$\dfrac{1}{4}$ page	$60	5
$\dfrac{1}{8}$ page	$35	12

Calculate the following.

a) the total number of pages of advertising sold

b) the total revenue from advertising

c) the average revenue per page of advertising sold

12. One week, Marjorie spent $\frac{1}{2}$ of her allowance on a music video, $\frac{1}{4}$ of her allowance on a T-shirt, and $\frac{1}{8}$ of her allowance on bus fares. She had $5 of her allowance left at the end of the week. How much was her allowance that week?

13. Add one pair of brackets to the left side of each equation to make the equation true.

a) $\frac{5}{2} \times \frac{3}{5} - \frac{2}{5} + \frac{1}{2} = 1$

b) $1\frac{1}{2} + 2\frac{1}{2} \div \frac{3}{4} - \frac{1}{8} = 5\frac{1}{2}$

c) $\frac{2}{3} - \frac{1}{6} + \frac{5}{6} \div \frac{16}{9} = \frac{3}{4}$

14. Here is a way of using four $\frac{1}{2}$s and the order of operations to write an expression that equals 2.

$$\frac{1}{2} \div \frac{1}{2} + \frac{1}{2} \div \frac{1}{2}$$

Use four $\frac{1}{2}$s and the order of operations to write expressions with each of the following values. Compare your expressions with your classmates' expressions.

a) 0   **b)** 1   **c)** $\frac{1}{4}$

d) 3   **e)** $\frac{1}{2}$ **f)** 4

g) $\frac{5}{8}$ **h)** $\frac{5}{4}$ **i)** $2\frac{1}{2}$

Extend

15. The mean of four fractions is $\frac{2}{3}$. Three of the fractions are $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{3}{4}$. What is the fourth fraction?

> **Literacy Link**
>
> The mean of a set of fractions is their sum divided by the number of fractions.
>
> The mean of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{8}$ is $\left(\frac{1}{4} + \frac{1}{2} + \frac{1}{8}\right) \div 3$, which equals $\frac{7}{24}$.

16. There are $1\frac{4}{9}$ times as many white notes as black notes on a full-sized piano keyboard. There are 88 notes altogether. Determine the number of white notes and the number of black notes.

17. Pedro's CDs are stored in three full racks of different sizes. The small rack holds $\frac{1}{2}$ as many CDs as the medium rack. The medium rack holds $\frac{1}{2}$ as many CDs as the large rack. There are 224 CDs altogether. How many are in each rack?

MATH LINK

About $\frac{1}{4}$ of the species of mammals that live in Canada can be found in the Taiga Shield ecozone. About 50 species of mammals can be found in this ecozone. How many species of mammals in Canada live outside the Taiga Shield ecozone?