6.5 Dividing Fractions and Mixed Numbers

Focus on...

After this lesson, you will be able to...

- divide two fractions or mixed numbers
- solve problems involving the division of fractions or mixed numbers

Russia and Canada are the two countries with the largest areas in the world. How are the fraction and the mixed number in the following two statements related?

The area of Canada is about $\frac{3}{5}$ of the area of Russia.

The area of Russia is about $1\frac{2}{3}$ times the area of Canada.

Explore the Math

How can you divide a fraction by a fraction?

Work with a partner.

1. When you divide 8 by 4, you determine how many 4s there are in 8. $8 \div 4 = 2$.

a) Show how you could determine how many $\frac{1}{2}$s there are in 3.

b) Copy and complete the equation $3 \div \frac{1}{2} = \blacksquare$.

2. a) Show how you could determine how many $\frac{1}{4}$s there are in 2.

b) Copy and complete the equation $2 \div \frac{1}{4} = \blacksquare$.

3. a) Show how you could determine how many $\frac{1}{4}$s there are in $\frac{3}{4}$.

b) Copy and complete the equation $\frac{3}{4} \div \frac{1}{4} = \blacksquare$.

4. a) Show how you could determine how many $\frac{1}{6}$s are in $\frac{2}{3}$.

b) Copy and complete the equation $\frac{2}{3} \div \frac{1}{6} = \blacksquare$.

5. a) Copy the table. Complete the divisions in the first column.

Division	Division With Equal Denominators
$\frac{3}{4} \div \frac{1}{4} = \blacksquare$	$\frac{3}{4} \div \frac{1}{4} = \blacksquare$
$\frac{2}{3} \div \frac{1}{3} = \blacksquare$	$\frac{2}{3} \div \frac{1}{3} = \blacksquare$
$\frac{8}{9} \div \frac{2}{9} = \blacksquare$	$\frac{8}{9} \div \frac{2}{9} = \blacksquare$
$\frac{2}{3} \div \frac{1}{6} = \blacksquare$	$\frac{4}{6} \div \frac{1}{6} = \blacksquare$
$\frac{1}{2} \div \frac{1}{12} = \blacksquare$	$\frac{6}{12} \div \frac{1}{12} = \blacksquare$
$\frac{3}{4} \div \frac{3}{8} = \blacksquare$	$\frac{6}{8} \div \frac{3}{8} = \blacksquare$

b) How are the divisions in the second column related to the divisions in the first column? Explain.

c) Complete the second column.

d) Write a rule for dividing two fractions using common denominators.

6. a) Copy the table. Complete the divisions in the first column.

Division	Multiplication
$\frac{3}{4} \div \frac{1}{4} = \blacksquare$	$\frac{3}{4} \times \frac{4}{1} = \blacksquare$
$\frac{2}{3} \div \frac{1}{3} = \blacksquare$	$\frac{2}{3} \times \frac{3}{1} = \blacksquare$
$\frac{8}{9} \div \frac{2}{9} = \blacksquare$	$\frac{8}{9} \times \frac{9}{2} = \blacksquare$
$\frac{2}{3} \div \frac{1}{6} = \blacksquare$	$\frac{2}{3} \times \frac{6}{1} = \blacksquare$
$\frac{1}{2} \div \frac{1}{12} = \blacksquare$	$\frac{1}{2} \times \frac{12}{1} = \blacksquare$
$\frac{3}{4} \div \frac{3}{8} = \blacksquare$	$\frac{3}{4} \times \frac{8}{3} = \blacksquare$

b) Complete the multiplications in the second column.

c) How are the multiplications in the second column related to the divisions in the first column? Explain.

d) Write a rule for dividing by a fraction using multiplication.

Strategies
Look for a Pattern

Reflect on Your Findings

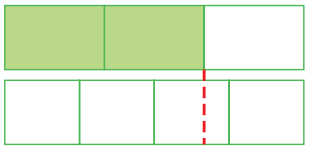
7. Which method do you prefer to use to divide a fraction by a fraction? Why?

Example 1: Divide Using Diagrams

Determine $\frac{2}{3} \div \frac{1}{4}$.

Solution

Use diagrams to determine how many $\frac{1}{4}$s are in $\frac{2}{3}$.



The diagram shows that the number of $\frac{1}{4}$s in $\frac{2}{3}$ is between 2 and 3.

When you divide a number by a proper fraction, the quotient is greater than the original number.
$\frac{8}{3} > \frac{2}{3}$

A common denominator for $\frac{1}{4}$ and $\frac{2}{3}$ is 12. So use a rectangle divided into twelfths.

In $\frac{8}{12}$, there are 2 whole groups of $\frac{3}{12}$, plus $\frac{2}{3}$ of another group.

$\frac{2}{3} \div \frac{1}{4} = 2\frac{2}{3}$ or $\frac{8}{3}$

Show You Know

Determine using diagrams.

a) $\frac{3}{4} \div \frac{1}{3}$ **b)** $1\frac{1}{4} \div \frac{3}{8}$ **c)** $\frac{1}{10} \div \frac{1}{5}$

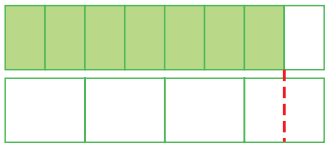
Example 2: Divide Using a Rule

Estimate and calculate.

a) $\frac{7}{8} \div \frac{1}{4}$ **b)** $2\frac{1}{2} \div 3\frac{3}{4}$

Solution

a) The diagram shows that the number of $\frac{1}{4}$s in $\frac{7}{8}$ is between 3 and 4.



You can calculate the quotient using either of the following methods.

reciprocal

- the multiplier of a number to give a product of 1
- $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$, because $\frac{3}{2} \times \frac{2}{3} = 1$
- the result of switching the numerator and denominator in a fraction

Method 1: Divide Using a Common Denominator

To divide fractions, write them with a common denominator and divide the numerators.

$\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \div \frac{2}{8}$

$= \frac{7}{2}$ or $3\frac{1}{2}$

$\frac{1}{4} = \frac{2}{8}$

Method 2: Divide Using Multiplication

To divide by a fraction, multiply by its reciprocal.

$\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \times \frac{4}{1}$

$= \frac{28}{8}$

$= \frac{7}{2}$ or $3\frac{1}{2}$

b) You can estimate the quotient by dividing the whole numbers closest to the mixed numbers.

$$2\frac{1}{2} \div 3\frac{3}{4} \approx 3 \div 4$$
$$\approx \frac{3}{4}$$

You can calculate the quotient using either of the following methods.

Method 1: Divide Using a Common Denominator

$$2\frac{1}{2} \div 3\frac{3}{4} = \frac{5}{2} \div \frac{15}{4}$$
$$= \frac{10}{4} \div \frac{15}{4}$$
$$= \frac{10}{15}$$
$$= \frac{2}{3}$$

Method 2: Divide Using Multiplication

$$2\frac{1}{2} \div 3\frac{3}{4} = \frac{5}{2} \div \frac{15}{4}$$
$$= \frac{5}{2} \times \frac{4}{15}$$
$$= \frac{20}{30}$$
$$= \frac{2}{3}$$

$\frac{4}{15}$ is the reciprocal of $\frac{15}{4}$.

When you divide a number by an improper fraction or mixed number, the quotient is less than the original number.
$\frac{2}{3} < 2\frac{1}{2}$

Show You Know

Estimate and calculate.

a) $\frac{4}{5} \div \frac{3}{10}$ **b)** $\frac{2}{9} \div \frac{5}{6}$ **c)** $3\frac{1}{6} \div 1\frac{2}{3}$

Example 3: Apply Division With Fractions

The baby teeth, or milk teeth, that develop in childhood are replaced by larger teeth as people mature. A full set of teeth for a child has $\frac{5}{8}$ as many teeth as a full set of teeth for an adult. There are 20 teeth in a full set for a child. How many teeth are there in a full set for an adult?

Solution

Divide 20 by $\frac{5}{8}$ to determine the number of adult teeth.

$$20 \div \frac{5}{8} = \frac{20}{1} \div \frac{5}{8}$$
$$= \frac{20}{1} \times \frac{8}{5}$$
$$= \frac{160}{5}$$
$$= 32$$

Instead of multiplying, you could divide using common denominators.
$20 \div \frac{5}{8} = \frac{20}{1} \div \frac{5}{8}$
$= \frac{160}{8} \div \frac{5}{8}$

There are 32 teeth in a full set for an adult.

Check:
Use multiplication to check the division.

$$\frac{5}{8} \times 32 = \frac{160}{8}$$
$$= 20$$

Did You Know?

A beaver, like other rodents, has only one set of teeth in its lifetime. A beaver's front teeth can grow by over a metre a year. The gnawing that the beaver does to cut trees and to eat wears down its teeth and keeps them the right length.

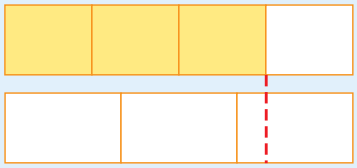
20 is $\frac{5}{8}$ of the number of adult teeth, so there are more than 20 adult teeth.
Dividing 20 by $\frac{5}{8}$ will result in a quotient greater than 20.
$20 \div \frac{5}{8} \approx 20 \div \frac{1}{2}$
≈ 40

Show You Know

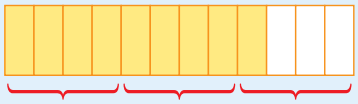
If one serving is $\frac{1}{6}$ of a tray of lasagna, how many servings are in three trays of lasagna?

Key Ideas

- You can estimate and determine the quotient of two fractions using diagrams.



$\frac{3}{4} \div \frac{1}{3}$ is between 2 and 3.



$\frac{3}{4} \div \frac{1}{3} = 2\frac{1}{4}$ or $\frac{9}{4}$

- You can estimate the quotient of two improper fractions or mixed numbers by dividing the whole numbers closest to them.

$$5\frac{1}{4} \div 3\frac{1}{2} \approx 5 \div 4$$
$$\approx \frac{5}{4} \text{ or } 1\frac{1}{4}$$

- To divide two fractions, you can write them with a common denominator and divide the numerators.

$$\frac{7}{10} \div \frac{3}{5} = \frac{7}{10} \div \frac{6}{10}$$
$$= \frac{7}{6} \text{ or } 1\frac{1}{6}$$

$$1\frac{3}{4} \div 2\frac{1}{2} = \frac{7}{4} \div \frac{5}{2}$$
$$= \frac{7}{4} \div \frac{10}{4}$$
$$= \frac{7}{10}$$

- To divide by a fraction, you can multiply by its reciprocal.

$$\frac{7}{10} \div \frac{3}{5} = \frac{7}{10} \times \frac{5}{3}$$
$$= \frac{35}{30}$$
$$= \frac{7}{6} \text{ or } 1\frac{1}{6}$$

$$1\frac{3}{4} \div 2\frac{1}{2} = \frac{7}{4} \div \frac{5}{2}$$
$$= \frac{7}{4} \times \frac{2}{5}$$
$$= \frac{14}{20}$$
$$= \frac{7}{10}$$

Communicate the Ideas

1. Anna divided $\frac{5}{6}$ by $\frac{1}{2}$ and stated that the quotient is $\frac{5}{12}$.

a) What was Anna's mistake?

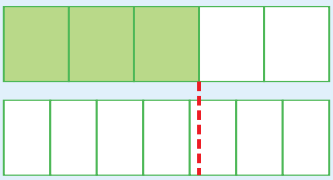
b) What is the correct quotient? Use a diagram to show how you know.

2. Mike carried out the division $\frac{3}{4} \div \frac{2}{3}$ as follows.

$$\frac{3}{4} \div \frac{2}{3} = \frac{4}{3} \times \frac{2}{3}$$
$$= \frac{8}{9}$$

Do you agree with Mike's method and answer? Explain.

3. Explain how the diagram shows that the quotient of $\frac{3}{5} \div \frac{1}{7}$ is between 4 and 5.



4. a) Does $2\frac{1}{2} \div 1\frac{1}{2}$ equal $1\frac{1}{2} \div 2\frac{1}{2}$?

b) If the quotients in part a) are not equal, how are they related?

Check Your Understanding

Practise

For help with #5 and #6, refer to Example 1 on page 224.

5. Determine each quotient using diagrams.

a) $\frac{5}{8} \div \frac{1}{4}$ **b)** $\frac{1}{4} \div \frac{1}{3}$

c) $1\frac{1}{2} \div \frac{2}{3}$ **d)** $2\frac{1}{3} \div \frac{5}{6}$

6. Use diagrams to determine each quotient.

a) $\frac{9}{10} \div \frac{1}{5}$ **b)** $\frac{1}{4} \div \frac{3}{8}$

c) $1\frac{2}{3} \div \frac{1}{2}$ **d)** $2\frac{3}{4} \div \frac{2}{3}$

For help with #7 to #10, refer to Example 2 on pages 224–225.

7. Divide using a common denominator.

a) $\frac{3}{5} \div \frac{9}{10}$ **b)** $1\frac{1}{2} \div \frac{5}{6}$ **c)** $3\frac{1}{3} \div 1\frac{5}{6}$

8. Divide using multiplication.

a) $\frac{5}{12} \div \frac{3}{4}$ **b)** $4\frac{1}{2} \div 1\frac{1}{4}$ **c)** $10 \div 2\frac{1}{2}$

9. Divide.

a) $\frac{3}{4} \div \frac{4}{5}$ **b)** $1\frac{2}{3} \div 2\frac{5}{6}$ **c)** $12 \div \frac{3}{4}$

10. Divide.

a) $1\frac{1}{12} \div 2\frac{1}{2}$ **b)** $\frac{8}{11} \div \frac{4}{5}$ **c)** $1\frac{3}{8} \div 2\frac{3}{4}$

Apply

For help with #11 to #13, refer to Example 3 on page 225.

11. In a comedy review, each performer has a $\frac{1}{4}$-h slot. How many performers are there in a 2-h show?

12. It takes $2\frac{1}{2}$ scoops of flour to make one cake. How many cakes do 15 scoops of flour make?

13. Three quarters of a can of apple juice fills six glasses. How many glasses will a whole can of apple juice fill?

14. An incandescent light bulb uses about $4\frac{1}{2}$ times as much energy as a compact fluorescent light bulb to produce the same amount of light. What fraction of the energy used by the incandescent bulb does the fluorescent light bulb use?

15. Shana and Zack painted their rooms using paint in cans of the same size. Shana used $1\frac{1}{2}$ cans of paint. Zack used $2\frac{3}{4}$ cans of paint. How many times as much paint did Zack use as Shana?

16. Of all the land on Earth, about $\frac{3}{10}$ is in Asia and about $\frac{3}{25}$ is in South America. How many times as big as South America is Asia?

17. The average wind speed in Calgary is $\frac{4}{5}$ of the average wind speed in Regina. The average wind speed in Calgary is 16 km/h. What is the average wind speed in Regina?

18. Use examples to explain your answer to each of the following.

a) Can the reciprocal of a proper fraction be a proper fraction?

b) Can the product of two proper fractions be greater than 1?

c) Can the quotient of two proper fractions be greater than 1?

19. a) The world's longest river is the Nile in Africa, with a length of 6825 km. This is about $1\frac{5}{8}$ times a long as the Mackenzie River, which is Canada's longest river.

a) How long is the Mackenzie River?

b) The Mackenzie River is about $2\frac{1}{10}$ times as long as the Columbia River. How long is the Columbia River?

20. Russia covers about $\frac{1}{30}$ of the Earth's surface. The area of Russia is about $1\frac{2}{3}$ times the area of Canada. What fraction of the Earth's surface does Canada cover?

21. Suppose a friend knows how to divide by whole numbers, but not by fractions.

a) How could you use the following pattern to show your friend how to calculate $4 \div \frac{1}{2}$?

$4 \div 8 = \frac{1}{2}$
$4 \div 4 = 1$
$4 \div 2 = 2$
$4 \div 1 = 4$
$4 \div \frac{1}{2} = \blacksquare$

b) Make up a pattern to show your friend how to calculate $9 \div \frac{1}{3}$.

22. Write a word problem that you can solve using the expression $3\frac{3}{4} \div 2\frac{1}{4}$.

Extend

23. It took Svend $9\frac{3}{4}$ min to ski up a slope on a cross-country ski trail and only $2\frac{1}{4}$ min to ski back down the same slope. How many times as fast did he ski down the slope as he skied up it?

24. The three largest islands in Canada are all north of the Arctic Circle. Baffin Island has about $2\frac{1}{3}$ times the area of Victoria Island. Baffin Island has about $2\frac{3}{5}$ times the area of Ellesmere Island. What fraction of the area of Victoria Island is the area of Ellesmere Island?

MATH LINK

The Prairies ecozone includes the Manitoba Plain and the grasslands of southwest Saskatchewan and southeast Alberta. The wettest part of this ecozone is the Manitoba Plain, which has an average annual precipitation of about 70 cm. This amount of precipitation is $2\frac{4}{5}$ of the amount in the dry grasslands. What is the average annual precipitation in these grasslands?

Did You Know?

The Prairies ecozone contains much of Canada's farmland, but it is vulnerable to droughts.