

6.4

Multiplying Improper Fractions and Mixed Numbers

Focus on...

After this lesson, you will be able to...

- multiply two improper fractions or mixed numbers
- solve problems involving the multiplication of improper fractions or mixed numbers



WWW Web Link

To find out more about provincial and territorial flags, go to www.mathlinks8.ca and follow the links. For which of the provincial and territorial flags does the width equal half the length?

The length of the flag of British Columbia is $1\frac{2}{3}$ times its width. How would you determine the length of a flag that is 90 cm wide?

Explore the Math

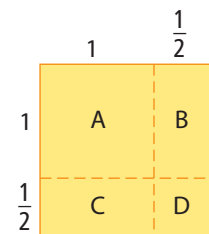
Did You Know?

The study of flags is called vexillology, after the Latin word for flag, *vexillum*.

How can you multiply two improper fractions or mixed numbers?

1. The diagram shows a way to model the multiplication $1\frac{1}{2} \times 1\frac{1}{2}$.

- a) What is the total area of the large square? Show your method. Express the total area as a mixed number in lowest terms.
- b) Write an equation to represent the multiplication.



2. Model each of the following multiplications using a similar model to the one used in #1. Express each total as a mixed number in lowest terms.

- a) $2\frac{1}{2} \times 2\frac{1}{2}$
- b) $1\frac{1}{4} \times 1\frac{1}{4}$
- c) $1\frac{1}{2} \times 1\frac{1}{4}$

Literacy Link

Mixed Numbers in Lowest Terms

A mixed number is in lowest terms when the fraction is in lowest terms. For example, $3\frac{4}{8}$ expressed in lowest terms is $3\frac{1}{2}$.

$$\begin{array}{c} \div 4 \\ \curvearrowright \\ \frac{4}{8} = \frac{1}{2} \\ \curvearrowleft \\ \div 4 \end{array}$$

3. a) Copy the table. Use your results from #1 and #2 to complete it. The first line is partially completed for you.

| Multiplication of Mixed Numbers | Product Expressed as a Mixed Number | Multiplication of Improper Fractions | Product Expressed as an Improper Fraction |
|------------------------------------|-------------------------------------|--------------------------------------|---|
| $1\frac{1}{2} \times 1\frac{1}{2}$ | | $\frac{3}{2} \times \frac{3}{2}$ | |
| $2\frac{1}{2} \times 2\frac{1}{2}$ | | | |
| $1\frac{1}{4} \times 1\frac{1}{4}$ | | | |
| $1\frac{1}{2} \times 1\frac{1}{4}$ | | | |

- b) Write a rule to multiply two improper fractions?
 c) How is your rule the same or different from the rule you developed for multiplying two proper fractions? Explain.

Literacy Link

Converting Improper Fractions and Mixed Numbers

Convert by using the denominator to decide the number of parts in one whole.

In $\frac{11}{4}$, one whole is $\frac{4}{4}$.

$$\frac{11}{4} = \frac{4}{4} + \frac{4}{4} + \frac{3}{4}$$

$$\frac{11}{4} = 2\frac{3}{4}$$

In $3\frac{2}{5}$, one whole is $\frac{5}{5}$.

$$3\frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5}$$

$$3\frac{2}{5} = \frac{17}{5}$$

Reflect on Your Findings

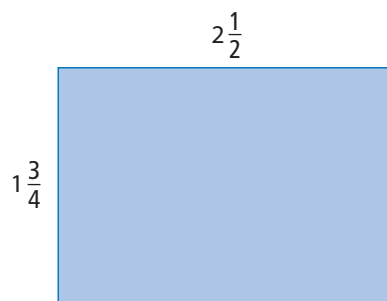
4. How can you multiply two improper fractions or mixed numbers by using a model or a rule?

Example 1: Multiply Mixed Numbers Using a Model

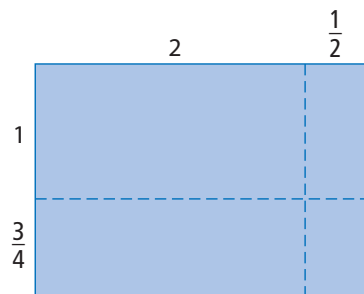
Determine $2\frac{1}{2} \times 1\frac{3}{4}$.

Solution

Draw a rectangle.



Draw a line segment to separate each dimension into a whole number and a proper fraction.



Strategies

Model It

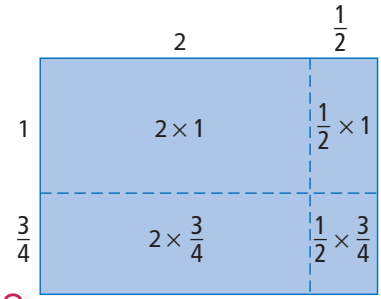
Literacy Link

A whole number can be written as a fraction with a denominator of 1.

For example, $2 = \frac{2}{1}$

Show the area of each of the four regions in the diagram.

$$\begin{aligned} 2 \times \frac{3}{4} &= \frac{2}{1} \times \frac{3}{4} \\ &= \frac{6}{4} \\ &= \frac{3}{2} \\ &= 1\frac{1}{2} \end{aligned}$$



Calculate the area of each region and add the areas.

$$2 \times 1 = 2 \quad 2 \times \frac{3}{4} = 1\frac{1}{2} \quad \frac{1}{2} \times 1 = \frac{1}{2} \quad \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$$2 + 1\frac{1}{2} + \frac{1}{2} + \frac{3}{8} = 4\frac{3}{8}$$

$$\text{So, } 2\frac{1}{2} \times 1\frac{3}{4} = 4\frac{3}{8}$$

Show You Know

Determine each product using a model.

a) $1\frac{3}{4} \times 1\frac{3}{4}$ b) $2\frac{1}{4} \times 2\frac{1}{4}$ c) $1\frac{1}{4} \times 1\frac{1}{3}$

Example 2: Multiply Mixed Numbers Using a Rule

Estimate and calculate $4\frac{1}{2} \times 2\frac{1}{3}$.

Solution

Estimate the product by multiplying the whole numbers closest to each mixed number.

$$\begin{aligned} 4\frac{1}{2} \times 2\frac{1}{3} &\approx 5 \times 2 \\ &\approx 10 \end{aligned}$$

To calculate $4\frac{1}{2} \times 2\frac{1}{3}$, write the mixed numbers as improper fractions.

$$4\frac{1}{2} \times 2\frac{1}{3} = \frac{9}{2} \times \frac{7}{3}$$

$$\begin{aligned} 4\frac{1}{2} &= \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{2}{2} + \frac{1}{2} \\ &= \frac{9}{2} \end{aligned} \quad \begin{aligned} 2\frac{1}{3} &= \frac{3}{3} + \frac{3}{3} + \frac{1}{3} \\ &= \frac{7}{3} \end{aligned}$$

To multiply the fractions, multiply the numerators and multiply the denominators.

$$\begin{aligned} \frac{9}{2} \times \frac{7}{3} &= \frac{63}{6} \\ &= \frac{21}{2} \\ &= 10\frac{1}{2} \end{aligned}$$

Write this fraction in lowest terms.

The answer is close to the estimate.

The product of two mixed numbers or improper fractions is greater than either of them.

$$10\frac{1}{2} > 4\frac{1}{2} \quad 10\frac{1}{2} > 2\frac{1}{3}$$

Show You Know

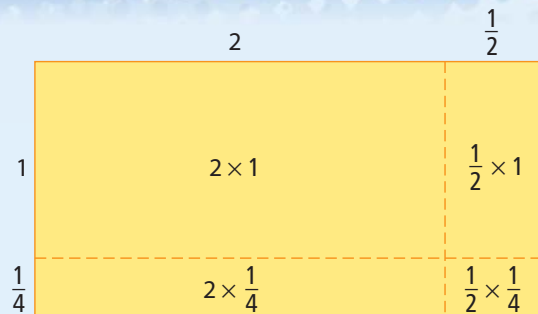
Estimate and calculate.

a) $1\frac{1}{10} \times 3\frac{1}{2}$ b) $1\frac{1}{4} \times 3\frac{2}{3}$

Key Ideas

- You can model the multiplication of two mixed numbers or improper fractions using partial areas of a rectangle.

$$\begin{aligned}2\frac{1}{2} \times 1\frac{1}{4} &= 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8} \\ &= 3\frac{1}{8}\end{aligned}$$



- You can estimate the product of two mixed numbers or improper fractions by multiplying the whole numbers closest to them.

$$\begin{aligned}3\frac{1}{4} \times 1\frac{3}{4} &\approx 3 \times 2 \\ &\approx 6\end{aligned}$$

- A rule for multiplying two mixed numbers is to express them as improper fractions and then multiply the numerators and multiply the denominators.

$$\begin{aligned}1\frac{2}{3} \times 2\frac{1}{5} &= \frac{5}{3} \times \frac{11}{5} \\ &= \frac{55}{15} \text{ or } \frac{11}{3}\end{aligned}$$

Communicate the Ideas

- Henri multiplied $2\frac{1}{2} \times 3\frac{1}{4}$ as follows: $2 \times 3 = 6$ and $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$, so $2\frac{1}{2} \times 3\frac{1}{4} = 6\frac{1}{8}$.
 - What mistake did Henri make?
 - What is the correct product?
- To express $4\frac{2}{3}$ as an improper fraction, Naomi determined the numerator by calculating $3 \times 4 + 2$.
 - Explain why Naomi's method works.
 - Use your explanation to write a rule for expressing a mixed number as an improper fraction. Test your rule.
 - Write a rule for expressing an improper fraction as a mixed number. Test your rule.

3. Moira multiplied $2\frac{1}{3} \times 2\frac{1}{2}$ as follows:

$$\begin{aligned}2\frac{1}{3} \times 2\frac{1}{2} &= \frac{7}{3} \times \frac{5}{2} \\ &= \frac{14}{6} \times \frac{15}{6} \\ &= \frac{210}{36} \\ &= \frac{35}{6} \\ &= 5\frac{5}{6}\end{aligned}$$

- a) Was her final answer correct?
b) How did she make the calculation longer than necessary

Check Your Understanding

Practise

4. Express each improper fraction as a mixed number.
a) $\frac{11}{3}$ b) $\frac{17}{6}$ c) $\frac{25}{2}$ d) $\frac{8}{5}$
5. Express each mixed number as an improper fraction.
a) $4\frac{3}{4}$ b) $2\frac{7}{8}$ c) $6\frac{1}{3}$ d) $3\frac{4}{7}$

For help with #6 and #7, refer to Example 1 on pages 217–218.

6. Use a model to determine each product.
a) $1\frac{1}{3} \times \frac{3}{4}$ b) $2\frac{1}{2} \times 1\frac{3}{5}$
c) $1\frac{1}{3} \times 1\frac{1}{2}$ d) $2\frac{1}{2} \times 2\frac{1}{4}$
7. Determine each product using a model.
a) $\frac{1}{2} \times 2\frac{1}{2}$ b) $2\frac{1}{3} \times 2\frac{1}{3}$
c) $1\frac{1}{2} \times 2\frac{1}{3}$ d) $1\frac{1}{5} \times 1\frac{1}{2}$

For help with #8 and #9, refer to Example 2 on page 218.

8. Estimate and calculate.
a) $\frac{4}{5} \times \frac{10}{7}$ b) $5 \times 3\frac{3}{4}$ c) $2\frac{1}{5} \times 1\frac{2}{3}$
9. Estimate and calculate.
a) $\frac{8}{3} \times \frac{11}{6}$ b) $2\frac{5}{6} \times 4$ c) $6\frac{1}{2} \times 3\frac{1}{2}$

Apply

10. Two and a half laps of a running track equal 1 km. How many laps equal 3 km?
11. Earth turns on its axis once every 24 h. How many hours does Earth take to complete $2\frac{1}{4}$ turns?
12. On a day in Winnipeg with $10\frac{1}{2}$ h of daylight, it was sunny for $\frac{1}{3}$ of that time. For how many hours was it sunny that day?

13. Alexa takes $\frac{1}{4}$ h to ride her bicycle to her friend's house. If Alexa walks instead, the trip takes her $2\frac{1}{2}$ times as long. How long does Alexa take to walk to her friend's house
- in hours?
 - in minutes?



14. In Eric's apartment, the living room is $1\frac{3}{4}$ times as long and $2\frac{1}{2}$ times as wide as the den. Eric is buying the same type of carpet for both rooms. How many times as much will the carpet cost for the living room as for the den?
15. Andreas has \$18. Bonnie has $1\frac{2}{3}$ times as much as Andreas. Cheryl has $1\frac{3}{5}$ times as much as Bonnie. How much money do they have altogether?

16. A corner store buys goods at the wholesale price and sells them for $\frac{7}{5}$ of the wholesale price. The wholesale price of a case of 12 cans of stew is \$15. For how much does the store sell one can of stew?
17. If you multiply a mixed number and a proper fraction, how does each value compare with the value of the product?
18. Create your own word problem that involves the multiplication of two mixed numbers. Make sure that you can solve your problem. Give your problem to a classmate to solve.

Extend

19. Describe each pattern. Then write the next three terms in each pattern.
- $4\frac{1}{3}, 2\frac{1}{6}, 1\frac{1}{12}, \frac{13}{24}, \dots$
 - $4, 6, 9, 13\frac{1}{2}, \dots$
20. Calculate.
- $4 \times 1\frac{1}{2} \times 2\frac{1}{2}$
 - $\frac{2}{3} \times 3\frac{1}{3} \times 4\frac{1}{2}$
 - $2\frac{3}{4} \times 1\frac{1}{3} \times 3\frac{1}{2}$
 - $1\frac{1}{6} \times 1\frac{2}{5} \times 2\frac{2}{7}$
21. Copy each equation. Use a mixed number to complete it.
- $1\frac{2}{3} \times \blacksquare = 2\frac{1}{2}$
 - $\blacksquare \times 2\frac{1}{6} = 2\frac{3}{5}$
 - $\blacksquare \times 1\frac{1}{4} = 3\frac{1}{8}$
 - $2\frac{1}{3} \times \blacksquare = 5\frac{5}{6}$

MATH LINK

The Hudson Plains ecozone contains most of Canada's wetlands. This ecozone covers about $\frac{1}{26}$ of the area of Canada. The Northern Arctic ecozone is one of the world's largest Arctic ecosystems. This ecozone is about $3\frac{9}{10}$ times as big as the Hudson Plains ecozone. What fraction of the area of Canada does the Northern Arctic ecozone cover?

