

6.3

Multiplying Proper Fractions

Focus on...

After this lesson, you will be able to...

- multiply two proper fractions
- solve problems involving the multiplication of two proper fractions



A two-toed sloth sleeps for 20 h per day. A chimpanzee sleeps $\frac{1}{2}$ that much. A horse sleeps $\frac{1}{2}$ as much as a chimpanzee. In a day, what fraction of the time that a two-toed sloth sleeps does a horse sleep? How do you know?

Explore the Math

Materials

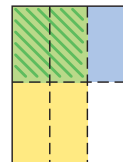
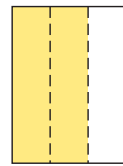
- six sheets of plain paper
- yellow and blue coloured pencils or crayons

How can you multiply two proper fractions?

1. You can determine $\frac{1}{2}$ of $\frac{2}{3}$ using paper folding.
 - Fold a rectangular piece of paper into thirds along its length. Open the paper and shade $\frac{2}{3}$ of it yellow.
 - Fold the paper in half across its width. Open the paper and shade half of it blue.

How does the model show $\frac{1}{2}$ of $\frac{2}{3}$?

Will the product be greater than or less than $\frac{2}{3}$?



2. Copy the table. Complete the table by using the method from #1. Do not write the products in lowest terms.

Multiplication	Product
$\frac{1}{2} \times \frac{2}{3}$	
$\frac{1}{3} \times \frac{1}{2}$	
$\frac{3}{4} \times \frac{1}{3}$	
$\frac{3}{4} \times \frac{2}{3}$	
$\frac{3}{4} \times \frac{3}{4}$	

3. For each row of the table, describe the relationship between
 a) the numerators b) the denominators
4. Use your answers from #3 to write a rule for multiplying two proper fractions.
5. a) Test your rule on the product $\frac{3}{4} \times \frac{1}{2}$.
 b) Check your answer to part a) by paper folding.



Reflect on Your Findings

6. a) How can you multiply two proper fractions by paper folding or using a rule?
 b) Which method do you prefer? Explain.

Example 1: Multiply Using Paper Folding

Determine $\frac{1}{2} \times \frac{3}{5}$.

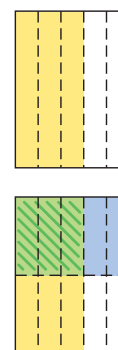
Solution

Fold a rectangular piece of paper into fifths along its length.
 Open the paper and shade three fifths of it yellow.
 Fold the paper in half across its width.

Open the paper and shade half of it blue.
 The folds make ten equal rectangles.
 Three of them are shaded both yellow and blue, so they appear green.

$$\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$$

The numerator in the product is the number of green rectangles.
 The denominator in the product is the total number of equal rectangles.



Show You Know

Determine each product using paper folding.

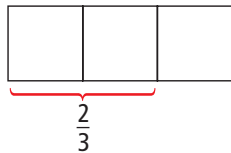
a) $\frac{1}{4} \times \frac{1}{2}$ b) $\frac{2}{3} \times \frac{2}{3}$

Example 2: Multiply Using Diagrams

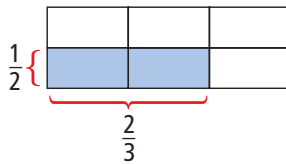
Determine $\frac{2}{3} \times \frac{1}{2}$.

Solution

Draw a rectangle. Draw line segments to cut its length into thirds.



Draw a line segment to cut the width of the rectangle into halves.



The diagram is like the result of paper folding.

Literacy Link

Understanding Common Denominators

For $\frac{1}{2}$ and $\frac{2}{3}$, a common denominator is 6, which is a common multiple of 2 and 3.

$$\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$$

Write $\frac{2}{6}$ in lowest terms.

$$\frac{2}{6} = \frac{1}{3}$$

$$\text{So, } \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

The product of two proper fractions is less than either of the fractions. You can check this relationship using common denominators.

$$\frac{2}{3} = \frac{4}{6} \quad \frac{1}{2} = \frac{3}{6}$$

$$\frac{2}{6} < \frac{4}{6} \quad \frac{2}{6} < \frac{3}{6}$$

Show You Know

Determine each product using diagrams.

a) $\frac{1}{2} \times \frac{1}{2}$ b) $\frac{1}{3} \times \frac{3}{4}$

Example 3: Multiply Using a Rule

Estimate and calculate $\frac{8}{15} \times \frac{5}{6}$.

Solution

Decide whether each fraction is closer to 0, $\frac{1}{2}$, or 1.



$$\frac{8}{15} \approx \frac{1}{2} \quad \frac{5}{6} \approx 1$$

Then estimate the product.

$$\begin{aligned} \frac{8}{15} \times \frac{5}{6} &\approx \frac{1}{2} \times 1 \\ &\approx \frac{1}{2} \end{aligned}$$

To multiply fractions, multiply the numerators and multiply the denominators.

$$\begin{aligned} \frac{8}{15} \times \frac{5}{6} &= \frac{40}{90} \\ &= \frac{4}{9} \end{aligned}$$

The answer seems reasonable, because it is close to the estimate of $\frac{1}{2}$.

Show You Know

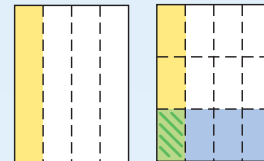
Estimate and calculate.

a) $\frac{3}{5} \times \frac{2}{9}$ b) $\frac{5}{6} \times \frac{4}{5}$

Key Ideas

- You can multiply two proper fractions using paper folding.

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$



- You can multiply two proper fractions using diagrams.

$$\frac{1}{2} \left\{ \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \color{green}{\square} & \color{green}{\square} & \color{green}{\square} & \square \\ \hline \end{array} \right. \quad \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$\underbrace{\hspace{10em}}_{\frac{3}{4}}$

- A rule for multiplying two proper fractions is to multiply the numerators and multiply the denominators.

$$\frac{3}{5} \times \frac{5}{6} = \frac{15}{30} \text{ or } \frac{1}{2}$$

- You can estimate the product of two proper fractions by first deciding whether each fraction is closer to 0, $\frac{1}{2}$, or 1.

$$\frac{3}{7} \approx \frac{1}{2} \quad \frac{4}{7} \approx \frac{1}{2} \quad \frac{3}{7} \times \frac{4}{7} \approx \frac{1}{4}$$

Communicate the Ideas

- Model $\frac{2}{3} \times \frac{1}{3}$ using manipulatives or diagrams.
 - Which method did you choose? Explain why you chose it.
- Brendan calculated $\frac{3}{5} \times \frac{2}{5}$ as follows:
$$\frac{3}{5} \times \frac{2}{5} = \frac{6}{5}$$
 - What mistake did he make?
 - How could you use estimation to show Brendan that he made a mistake?
 - What is the correct product?

Check Your Understanding

Practise

For help with #3 and #4, refer to Examples 1 and 2 on pages 211–212.

- Determine each product using paper folding or diagrams.
 - $\frac{5}{6} \times \frac{1}{2}$
 - $\frac{3}{4} \times \frac{5}{6}$
- Use paper folding or diagrams to determine each product.
 - $\frac{1}{4} \times \frac{2}{3}$
 - $\frac{7}{10} \times \frac{1}{2}$

For help with #5 and #6, refer to Example 3 on page 213.

- Estimate and calculate each product. Express your answer in lowest terms.
 - $\frac{3}{8} \times \frac{2}{3}$
 - $\frac{3}{7} \times \frac{1}{6}$
 - $\frac{3}{4} \times \frac{3}{4}$
- Estimate and calculate each product. Express your answer in lowest terms.
 - $\frac{2}{5} \times \frac{4}{5}$
 - $\frac{7}{8} \times \frac{4}{5}$
 - $\frac{3}{4} \times \frac{4}{9}$

Apply

- Tamar had $\frac{1}{2}$ of an apple pie in her refrigerator. She ate $\frac{1}{4}$ of this piece of pie. What fraction of a whole pie did she eat?
- Marius spends $\frac{1}{3}$ of his time sleeping. While he is asleep, he dreams for $\frac{1}{4}$ of the time.
 - For what fraction of his time is Marius dreaming?
 - For how many hours a day is Marius dreaming?
- About $\frac{1}{20}$ of the people in the world live in Canada or the United States. Of the people who live in Canada or the United States, about $\frac{1}{10}$ live in Canada. What fraction of the people in the world live in Canada?

10. At the age of four, the average person is about $\frac{3}{5}$ as tall as they will be as an adult. At birth, the average person is about $\frac{1}{2}$ as tall as they will be at the age of four. For the average person, what fraction is their height at birth of their height as an adult?

11. When the Summer Olympic and Paralympic Games were held in Athens, Greece, paralympic athletes won $\frac{6}{7}$ of Canada's total medals. Of the medals that Canadian paralympic athletes won, $\frac{7}{18}$ were gold medals.

a) What fraction of Canada's total medals were gold medals won by paralympic athletes?

b) Canada won a total of 84 medals. How many gold medals did Canadian paralympic athletes win?



12. Write a word problem that you can solve using the expression $\frac{3}{4} \times \frac{1}{2}$.

Extend

13. For a standard deck of 52 playing cards, the probability of randomly drawing a red card is $\frac{1}{2}$. The probability of randomly drawing a face card (jack, queen, or king) is $\frac{12}{52}$. What is the probability of randomly drawing a face card that is red?

14. Calculate. Express the product in lowest terms.

a) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

b) $\frac{2}{3} \times \frac{1}{4} \times \frac{2}{5}$

c) $\frac{3}{4} \times \frac{3}{4} \times \frac{2}{9}$

d) $\frac{5}{6} \times \frac{3}{8} \times \frac{7}{10}$

15. Copy each equation. Complete it using a fraction in lowest terms.

a) $\frac{\blacksquare}{\blacksquare} \times \frac{1}{2} = \frac{5}{16}$

b) $\frac{\blacksquare}{\blacksquare} \times \frac{3}{7} = \frac{1}{3}$

c) $\frac{2}{3} \times \frac{\blacksquare}{\blacksquare} = \frac{1}{2}$

d) $\frac{3}{4} \times \frac{\blacksquare}{\blacksquare} = \frac{5}{8}$

16. Use the sum and the product of two fractions to identify the fractions.

a) sum $\frac{1}{2}$; product $\frac{1}{16}$

b) sum $\frac{5}{6}$; product $\frac{1}{6}$

c) sum $\frac{2}{3}$; product $\frac{1}{12}$

MATH LINK

The area of British Columbia is about $\frac{1}{10}$ of the area of Canada. The Pacific Maritime ecozone covers about $\frac{1}{5}$ of the area of British Columbia. What fraction of the area of Canada does the Pacific Maritime ecozone cover?

