6.1 Multiplying a Fraction and a Whole Number

Focus on...

After this lesson, you will be able to...

- ❑ multiply a fraction and a whole number
- ❑ solve problems involving the multiplication of a fraction and a whole number

Chess is one of the most popular board games. It has been estimated that about $\frac{1}{5}$ of adult Canadians play chess at least once a year. The chess board shown has black and white squares. What fraction of the total number of squares are black? If you were told the total area of all the squares, how could you determine the total area of the black squares?

Explore the Math

Materials

- pattern blocks

Literacy Link

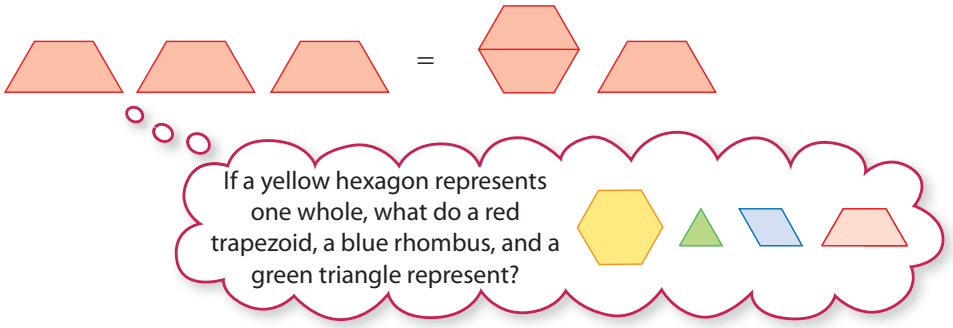
Understanding Multiplication

The product of 4 and 2 is 8, because $4 \times 2 = 8$.

The equation $4 \times 2 = 8$ means that 4 groups of 2 make 8. You can also think of 4×2 as the repeated addition $2 + 2 + 2 + 2$.

How can you model the multiplication of a fraction and a whole number?

1. a) How do the pattern blocks model an addition? Describe it.



b) How do the pattern blocks also model a multiplication? Describe it.

c) Work with a partner to explore other manipulatives you could use to model the multiplication.

2. a) Work with a partner to explore how you could use diagrams to model $4 \times \frac{1}{6}$.

b) Write an equation to represent your model.

3. a) Model $2 \times \frac{4}{3}$ using the method of your choice.

b) Write an equation to represent your model.

Reflect on Your Findings

4. a) Share your models with your classmates.

b) Suggest other manipulatives or diagrams you could use. How would you use them?

Example 1: Multiply Using A Model

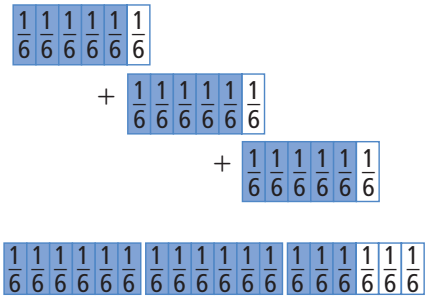
Determine $3 \times \frac{5}{6}$. Express the product in lowest terms.

Solution

You can express the multiplication as a repeated addition.

$3 \times \frac{5}{6} = \frac{5}{6} + \frac{5}{6} + \frac{5}{6}$

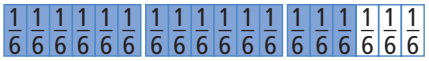
Model the fractions using fraction strips.

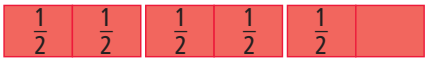


Count the shaded parts of the strips.

$\frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{15}{6}$

Write the product in lowest terms.

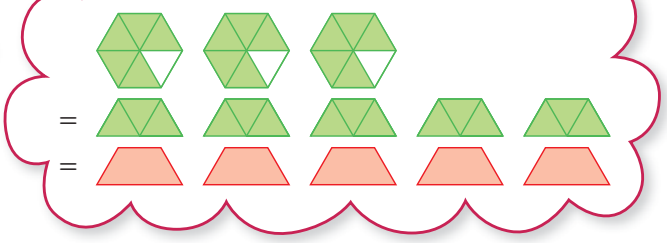




$\frac{15}{6} = \frac{5}{2}$

So, $3 \times \frac{5}{6} = \frac{5}{2}$.

You could model with pattern blocks instead of fraction strips.



You could divide the numerator and denominator by a common factor.

$\frac{15}{6} = \frac{5}{2}$

÷ 3 ÷ 3

Literacy Link

Classifying Fractions

In a *proper fraction*, such as $\frac{1}{2}$ or $\frac{5}{6}$, the denominator is greater than the numerator.

In an *improper fraction*, such as $\frac{5}{2}$ or $\frac{4}{3}$, the numerator is greater than the denominator.

A *mixed number*, such as $1\frac{1}{4}$ or $4\frac{3}{5}$, includes a whole number and a proper fraction.

Strategies

Model It

ME

The product of a natural number and a proper fraction is less than the natural number.

$\frac{5}{2} < 3$

Show You Know

Determine each product using models. Express the product in lowest terms.

a) $2 \times \frac{5}{6}$ **b)** $4 \times \frac{2}{3}$

Example 2: Multiply Using a Diagram

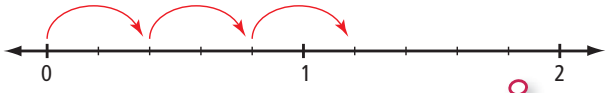
Determine $3 \times \frac{2}{5}$. Express the product in lowest terms.

Strategies
Draw a Diagram

Solution

$3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

Model the fractions using a number line.



$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5}$

You could draw rectangles instead of using a number line.

The answer is already in lowest terms.

So, $3 \times \frac{2}{5} = \frac{6}{5}$.

Show You Know

Determine each product using a diagram. Express the product in lowest terms.

a) $2 \times \frac{3}{2}$ **b)** $4 \times \frac{5}{8}$

Example 3: Apply Multiplication With Fractions

A spider has eight legs. An ant has $\frac{3}{4}$ as many legs as a spider. How many legs does an ant have?

Literacy Link
In mathematics, the word ***of*** often indicates multiplication.

Solution

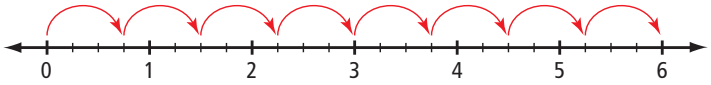
An ant has $\frac{3}{4}$ of the number of legs that a spider has.

$\frac{3}{4}$ of 8 means $\frac{3}{4} \times 8$.

Multiplying $\frac{3}{4} \times 8$ gives the same answer as multiplying $8 \times \frac{3}{4}$.

Determine $8 \times \frac{3}{4}$.

Model the multiplication as a repeated addition on a number line.



The result is 6. So, $8 \times \frac{3}{4} = 6$.

An ant has six legs.

Literacy Link

Commutative Property

The *commutative property* states that $a \times b = b \times a$.

M E

An ant has $\frac{3}{4}$ as many legs as a spider, so an ant has fewer legs than a spider.

$6 < 8$

Show You Know

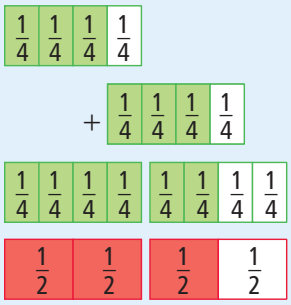
Jenelle is making a recipe that calls for six scoops of flour. She wants to make only $\frac{2}{3}$ of the recipe. How many scoops will she need to use?

Key Ideas

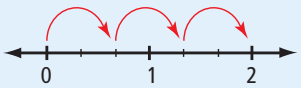
- You can show the multiplication of a fraction and a whole number using models and diagrams.

$3 \times \frac{1}{6} = \frac{1}{2}$

$2 \times \frac{3}{4} = \frac{3}{2}$



$3 \times \frac{2}{3} = 2$

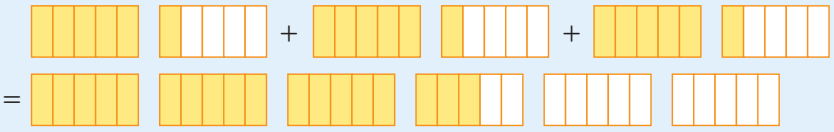


- Multiplying a fraction and a whole number in either order gives the same result.

$10 \times \frac{2}{5} = 4$ $\qquad$ $\frac{2}{5} \times 10 = 4$

Communicate the Ideas

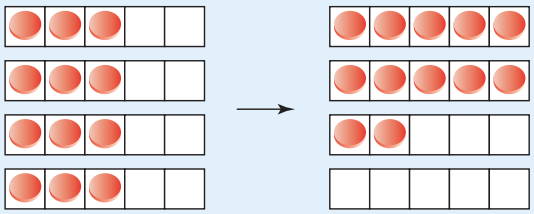
1. The diagram models $3 \times \frac{6}{5}$.



a) What equation does the diagram represent?

b) If a hexagon represents one whole, could you use pattern blocks to model the same multiplication? Explain.

2. Makoto found his own way to model $4 \times \frac{3}{5}$ by using counters on grids.



a) Why did he use 5-by-1 grids?

b) Why did he use four grids?

c) How does Makoto's model show the product?

3. Nadine said that she had her own method for determining $4 \times \frac{3}{5}$. She first multiplied 4 and 3 to get 12. She then wrote the product as $\frac{12}{5}$. Do you agree with Nadine's method for multiplying a whole number and a fraction? Explain using other examples.

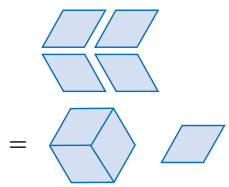
Check Your Understanding

Practise

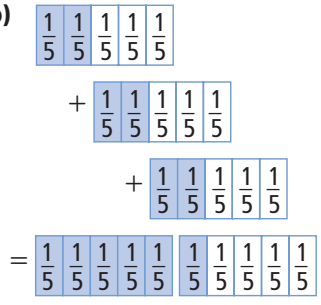
For help with #4 to #7, refer to Examples 1 and 2 on pages 199–200.

4. What equation does each model represent? For pattern blocks, assume that a hexagon represents one whole.

a)

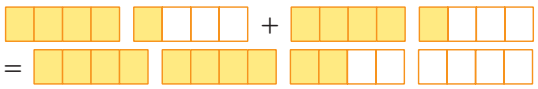


b)

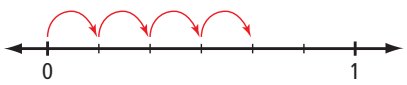


5. What equation does each diagram represent?

a)



b)



6. Determine each product using manipulatives or diagrams.

a) $4 \times \frac{1}{2}$

b) $3 \times \frac{7}{10}$

c) $5 \times \frac{2}{3}$

d) $3 \times \frac{3}{8}$

7. Determine each product.

a) $3 \times \frac{1}{8}$ **b)** $6 \times \frac{1}{4}$

c) $2 \times \frac{6}{5}$ **d)** $2 \times \frac{4}{3}$

Apply

For help with #8 to #9, refer to Example 3 on page 200.

8. The width of a Canadian flag is $\frac{1}{2}$ of its length. What is the width of a Canadian flag that is 4 m long?

9. A minibus that seats 12 people is $\frac{3}{4}$ full. How many people are seated in the minibus?

10. a) What fraction of the surface area of a cube is the area of one face?

b) What is the area of each face of a cube of surface area 6 cm^2?

11. Ron's car uses 12 L of gasoline per 100 km of highway driving. Asma's car uses only $\frac{5}{6}$ as much fuel. How much fuel does Asma's car use per 100 km of highway driving?

12. Nunavut covers about $\frac{1}{5}$ of the area of Canada. The area of Canada is about ten million square kilometres. What is the approximate area of Nunavut?

13. Suppose a friend knows how to multiply whole numbers, but not fractions.

a) How could you use the following pattern to show your friend how to calculate $\frac{1}{2} \times 10$?

$4 \times 10 = 40$
$2 \times 10 = 20$
$1 \times 10 = 10$
$\frac{1}{2} \times 10 = \blacksquare$

b) Make up a pattern to show your friend how to calculate $\frac{1}{3} \times 9$.

14. Write a word problem that you can solve using the expression $\frac{1}{4} \times 8$.

Extend

15. There are 30 students in a class. Four fifths of them have brown eyes. How many students have brown eyes?

16. The perimeter of an isosceles triangle is 15 cm. The shortest side equals $\frac{1}{5}$ of the perimeter. What are the side lengths of the triangle?

17. A ball dropped to the ground bounces back to $\frac{2}{3}$ of its previous height. If the ball is dropped straight down from a height of 81 cm, how far does it travel altogether by the time it hits the ground for the fifth time?

MATH LINK

A quarter of Canada's 20 ecozones are marine ecozones, which include parts of oceans. The rest of Canada's ecozones are terrestrial ecozones. They include parts of the land, and may contain rivers, lakes, and wetlands.

a) How many marine ecozones does Canada have?

b) How many terrestrial ecozones does Canada have?