

# 4.2

## Fractions, Decimals, and Percents

### Focus on...

After this lesson, you will be able to...

- convert between fractions, decimals, and percents



The period was not bad with 90% of the shots saved by the home team goalie. The second period saw 150% as many shots on goal, yet an amazing 0.9333 save performance held the home team in the game. But, how many ways can you spell disaster? In the third period, the home team goaltender let in two easy goals for a dismal  $66\frac{2}{3}\%$  of shots on goal stopped.

Sports commentators often use statistics to report on the performance of a goalie. Commentators often change the way the information is presented to make it sound more interesting.

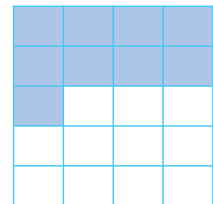
How did the sports commentator use the information from the following table in the report on the goalie's performance?

| Goalie Statistics |               |       |               |                   |
|-------------------|---------------|-------|---------------|-------------------|
| Period            | Shots on Goal | Saves | Goals Against | Save Percent      |
| 1                 | 10            | 9     | 1             | 90%               |
| 2                 | 15            | 14    | 1             | $93\frac{1}{3}\%$ |
| 3                 | 6             | 4     | 2             | $66\frac{2}{3}\%$ |

### Explore the Math

#### How are percents related to fractions and decimals?

1. a) What fraction of this figure is shaded?  
 b) Rewrite your fraction with a denominator of 100.  
 c) Express the fraction shaded as a decimal.  
 d) What percent of the figure is shaded?



2. Suppose you want to shade one half as many sections as in #1. Show the area that will be shaded on a new diagram. How much of the diagram will you shade? Express your answer as a fraction, a decimal, and a percent.

3. Suppose you want to shade three times as many sections as in #1. If one large square represents one whole, how many squares will you need to draw to show this situation? How many squares will you shade? Express your answer as a fraction, a decimal, and a percent.

## Reflect on Your Findings

4. a) How are the decimal, percent, and fraction representations of a number the same? How are they different?  
 b) Which representations do you prefer to work with? Why?

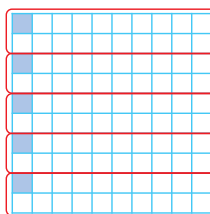
## Example 1: Convert Fractions to Decimals and Percents

Convert each fraction to a decimal and a percent.

a)  $\frac{1}{20}$       b)  $\frac{71}{200}$       c)  $\frac{9}{8}$

### Solution

- a) Percent means out of 100. So,  $\frac{1}{20} = \frac{x}{100}$ .  
 You could represent this using a hundred grid.  
 5 of 100 squares are coloured.  
 So,  $\frac{1}{20} = \frac{5}{100}$ . That is 5% or 0.05.  
 Sometimes you interpret  $\frac{1}{20}$  as  $1 \div 20 = 0.05$ .  
 0.05 can be expressed as 5%.

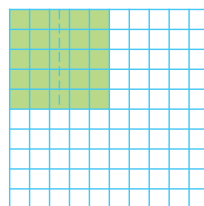


Is  $\frac{1}{20}$  greater than or less than one whole? Will the percent be greater than or less than 100%? **M E**

- b)  $\frac{71}{200} = \frac{x}{100}$   
 $x = 35.5$       How do you know  $x = 35.5$ ?
- That is 35.5% or 0.355.  
 You could interpret  $\frac{71}{200}$  as  $71 \div 200 = 0.355$ .  
 0.355 can be expressed as 35.5%.

Is  $\frac{71}{200}$  greater than or less than one whole? Will the percent be greater than or less than 100%? **M E**

- c)  $\frac{9}{8}$  can be expressed as  $\frac{8}{8} + \frac{1}{8} = 1 + \frac{1}{8}$ .  
 One whole represents 100%.  
 You know that  $\frac{1}{4}$  represents 25%. So,  $\frac{1}{8}$  represents 12.5%.  
 $\frac{9}{8}$  can be expressed as  $100\% + 12.5\% = 112.5\%$ .  
 You could also interpret  $\frac{9}{8}$  as  $9 \div 8 = 1.125$ .  
 1.125 can be expressed as 112.5%.



Is  $\frac{9}{8}$  greater than or less than one whole? Will the percent be greater than or less than 100%? **M E**

## Show You Know

Convert each fraction to a decimal and a percent.

a)  $\frac{3}{40}$

b)  $\frac{171}{300}$

c)  $\frac{88}{50}$

## Example 2: Convert Decimals to Percents and Fractions

Convert each decimal to a percent and a fraction.

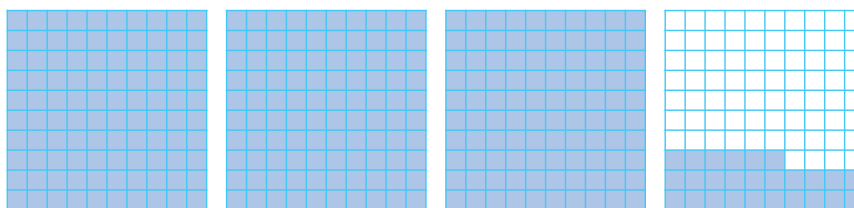
a) 3.26

b) 0.125

c) 0.0032

### Solution

a) Use hundred grids.



$3.26 = 3$  full hundred grids plus 26 squares

That is  $\frac{326}{100} = 326\%$ .

$3.26 = 3\frac{26}{100}$  or  $3\frac{13}{50}$ .

Since 13 is a prime number,  $3\frac{13}{50}$  is in lowest terms.

$\frac{326}{100}$  can also be expressed as  $\frac{163}{50}$  or  $3\frac{13}{50}$  in lowest terms.

What would you divide into both the numerator and denominator of  $\frac{326}{100}$  to get  $\frac{163}{50}$ ?

b)  $0.125 = \frac{125}{1000}$  since the 5 is in the thousandths place.

$0.125$  can also be expressed as  $\frac{12.5}{100}$  or  $12.5\%$ .

What factors of 125 divide evenly into 1000?

$\frac{125}{1000} = \frac{1}{8}$

How do you know  $\frac{125}{1000}$  and  $\frac{12.5}{100}$  are equivalent?

c)  $0.0032 = \frac{32}{10\,000}$  since the 2 is in the ten thousandths place.

$0.0032$  can also be written as  $\frac{0.32}{100}$  or  $0.32\%$ .

What factors of 32 divide evenly into 10 000?

$\frac{32}{10\,000} = \frac{2}{625}$

How do you know that  $\frac{32}{10\,000}$  and  $\frac{0.32}{100}$  are equivalent?

## Show You Know

Convert each decimal to a percent and a fraction.

a) 0.0064

b) 0.268

c) 5.98

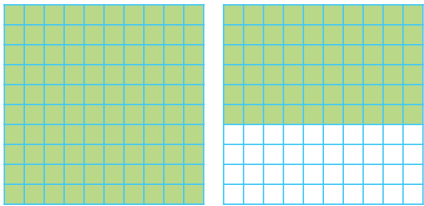
### Example 3: Convert Percents to Fractions and Decimals

Convert each percent to a decimal and a fraction.

- a) 160%      b) 0.35%      c)  $25\frac{3}{5}\%$

#### Solution

- a) You could represent 160% using hundred grids.



$$\frac{100}{100} + \frac{60}{100} = \frac{160}{100}$$

$\frac{160}{100}$  is equivalent to  $\frac{16}{10}$  or  $\frac{8}{5}$ .

You can interpret  $\frac{160}{100}$  as  $160 \div 100 = 1.6$ .

So, 160% can be expressed as 1.6,  $\frac{16}{10}$ , or  $\frac{8}{5}$ .

How do you know  $\frac{160}{100}$  and  $\frac{8}{5}$  are equivalent?

Is 160% greater than or less than one whole?

- b) Percent means out of 100. So, 0.35% can be written as  $\frac{0.35}{100}$ .

You can interpret  $\frac{0.35}{100}$  as  $0.35 \div 100 = 0.0035$ .

$0.0035 = \frac{35}{10\,000}$ , since the 5 is in the ten thousandths place.

$\frac{35}{10\,000}$  can be written in lowest terms as  $\frac{7}{2000}$ .

What factors of 35 divide evenly into 10 000?

Is 0.35% greater than or less than  $\frac{1}{100}$ ?

- c)  $25\frac{3}{5}\%$  can be expressed as  $25\% + \frac{3}{5}\%$ .

25% is 0.25 or  $\frac{1}{4}$ . You can interpret  $\frac{3}{5}$  as  $3 \div 5 = 0.6$ .

$\frac{3}{5}\%$  would be  $0.6 \div 100 = 0.006$ .

$$\begin{aligned} \text{So, } 25\frac{3}{5}\% &= 0.25 + 0.006 \\ &= 0.256 \end{aligned}$$

That is the same as  $\frac{256}{1000}$ .

$25\frac{3}{5}\%$  can be expressed as 0.256 or  $\frac{256}{1000}$ .

What is  $\frac{256}{1000}$  in lowest terms? Show your thinking.

Is  $25\frac{3}{5}\%$  greater than or less than  $\frac{1}{4}$ ?  
Is  $\frac{256}{1000}$  greater than or less than  $\frac{1}{4}$ ? How do you know?

#### Show You Know

Convert each percent to a decimal and a fraction.

- a) 750%      b) 0.3%      c)  $12\frac{3}{4}\%$

### Example 4: Determine a Percent

For the past century, the north magnetic pole has been drifting across the Canadian Arctic. Prior to the 1970s, the magnetic pole was drifting at an average speed of 10 km/year. Since the 1970s, the speed at which the magnetic pole has been drifting has increased to about 50 km/year. The circumference of Earth is approximately 40 000 km.

- What percent is the current speed of the original speed?
- The circumference of Earth is approximately 40 000 km. At 50 km/year, what percent of Earth's circumference will the pole drift in one year?

### Solution

- a)** The current speed is 50 km/year.  
The original speed is 10 km/year.

Divide to find what percent the current speed is of the original speed.

$$\frac{50}{10} = 5$$

Percent means out of 100. So,  $5 = \frac{500}{100}$ .

$$\text{So, } 5 = 500\%$$

The current speed is 500% of the original speed.

- b)** The circumference of Earth is 40 000 km.  
The distance the pole drifts in one year is 50 km.  
The amount of Earth's circumference travelled in one year is represented by

$$\frac{50}{40\,000} = \frac{1}{800}$$

$$= 0.00125$$

$$0.00125 = 0.125\%$$

At 50 km/year, the pole will drift 0.125% or  $\frac{1}{8}\%$  of Earth's circumference in one year.

0.125 is equivalent to the fraction  $\frac{1}{8}$ .

### Show You Know

Suppose that the speed at which the pole is drifting increased to 75 km/year.

- What percent is 75 km/year of the original speed?
- At 75 km/year, what percent of 40 000 km would the pole drift in one year?

## Key Ideas

- Fractions, decimals, and percents can be used to represent numbers in various situations.
- Percents can be written as fractions and as decimals.

$$\begin{array}{lll} \frac{1}{2}\% = 0.5\% & 150\% = \frac{150}{100} & 42\frac{3}{4}\% = 42.75\% \\ 0.5\% = \frac{0.5}{100} & = 1.5 \text{ or } 1\frac{1}{2} & 42.75\% = \frac{42.75}{100} \\ = 0.005 & & = 0.4275 \end{array}$$

## Communicate the Ideas

1. Kaitlyn and Jordan are converting 0.003 to a percent. Who is correct? Show how you know.

Kaitlyn:  
 $0.003 = 3\%$

Jordan:  
 $0.003 = 0.3\%$

2. Which number does not have the same value as the other three? Explain your reasoning.

$$\frac{12}{5} \quad 2.4 \quad 250\% \quad \frac{60}{25}$$

3. Teammates Mark and Jonas are discussing the outcome of a game. Mark says their team scored 500% as many goals as the other team and Jonas says they scored five times as many goals as the other team. Can they both be correct? Explain how you know.

## Check Your Understanding

### Practise

For help with #4 and #5, refer to Example 1 on page 131.

4. Convert each fraction to a decimal and a percent.
- a)  $\frac{1}{250}$       b)  $\frac{81}{200}$       c)  $\frac{7}{5}$

5. Rewrite each fraction as a decimal and a percent.

a)  $\frac{51}{30}$       b)  $\frac{21}{200}$       c)  $\frac{3}{500}$

For help with #6 and #7, refer to Example 2 on page 132.

6. Convert each decimal to a percent and a fraction.
- a) 0.0072      b) 0.548      c) 3.45

7. Change each decimal to a percent and a fraction.

- a) 0.256      b) 0.0005      c) 6.5

For help with #8 and #9, refer to Example 3 on page 133.

8. Convert each percent to a decimal and a fraction.

- a) 248%      b) 0.56%      c)  $75\frac{3}{4}\%$

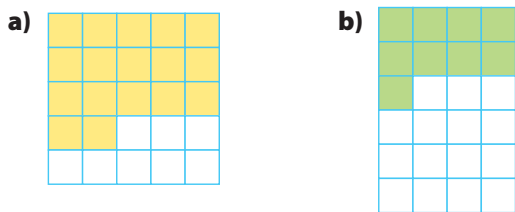
9. Express each percent as a decimal and a fraction.

- a)  $5\frac{9}{10}\%$       b) 550%      c) 0.8 %

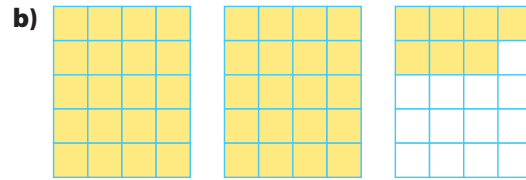
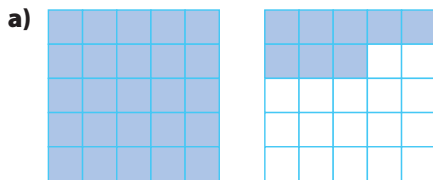
10. Copy and complete the following table. The first row is completed for you.

| Percent  | Fraction          | Decimal |
|----------|-------------------|---------|
| 165%     | $\frac{165}{100}$ | 1.65    |
| a) 230%  |                   |         |
| b) 0.38% |                   |         |
| c) 19.9% |                   |         |

11. Express the shaded portion of each diagram as a fraction, a decimal, and a percent.



12. If one completely shaded grid represents one whole, express the shaded portion of each diagram as a fraction, a decimal, and a percent.



### Apply

For help with #13 and #14, refer to Example 4 on page 134.

13. Several years ago Claire bought the first issue of a popular comic book for \$10. At a recent auction, it sold for \$200. What percent is the new value of the comic book of the price several years ago?

14. A snack contains 0.9 g of fat. Suppose that in one day, Shaun consumed a total of 40 g of fat, including the snack. What percent of Shaun's total fat consumption does the snack represent? What is this percent as a decimal and as a fraction?

15. Use hundred grids to help place the following numbers in ascending order.  $145\%$ ,  $\frac{5}{8}\%$ , 1.32, 0.65, 33.5%, 0.6%

16. A miner found 12 g of gold in a 2700-g sample of ore. What percent of the sample is gold, to the nearest tenth of a percent? What is the percent as a repeating decimal and as a fraction in lowest terms?

### Literacy Link

A repeating decimal contains a digit or group of digits that repeat forever. You can write a repeating decimal using bar notation.

$$0.333\ 33\dots = 0.\overline{3} \quad 0.454\ 545\dots = 0.\overline{45}$$

17. A fundraising coordinator is preparing an advertising flyer for an upcoming event. She wants to use either a fraction or a decimal number to represent each of the percents in the following statements. Decide whether a fraction or a decimal number is better and rewrite each statement using your chosen representation. Justify your choices.

- a) Ticket sales are 130% of what they were at this time last year.
- b) We are already at  $60\frac{1}{2}\%$  of our target and we just started!
- c) We have managed to cut our costs by 0.75%.

18. A fisheries worker recorded the following species and numbers of fish passing by a fish counter. Copy and complete the following table.

| Species   | Number | Percent of Total | Fraction of Total | Decimal Equivalent |
|-----------|--------|------------------|-------------------|--------------------|
| Chinook   | 143    |                  |                   |                    |
| Coho      | 122    |                  |                   |                    |
| Steelhead | 2      |                  |                   |                    |

19. Over five years, the circulation of a magazine increased from 25 000 copies to 150 000 copies. What percent is the new circulation of the circulation five years ago? What is this percent as a decimal and as a fraction?

20. Kim's resting heart rate was 75 beats per minute. A trainer advised Kim to have a portion of her workout at 90 beats per minute and a portion at 125 beats per minute, but not to exceed 150 beats per minute. Express each heart rate compared to the resting heart rate as a percent, a fraction, and a decimal.



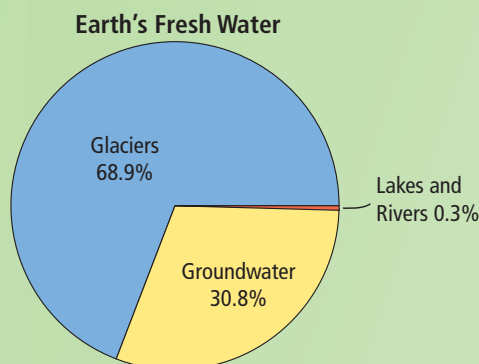
**Extend**

21. Copy and complete the first three rows of the table. Use the patterns in the first three rows to complete the last two rows.

| Percent | Decimal | Fraction      |
|---------|---------|---------------|
| a) 1000 |         |               |
| b)      | 5.00    |               |
| c)      |         | $\frac{5}{2}$ |
| d)      |         |               |
| e)      |         |               |

**MATH LINK**

Represent the percents shown in the circle graph in two other ways.



**Did You Know?**

In 2002, NASA launched two satellites to measure groundwater amounts from space! These satellites use gravity to weigh Earth's groundwater.