

# 3.5

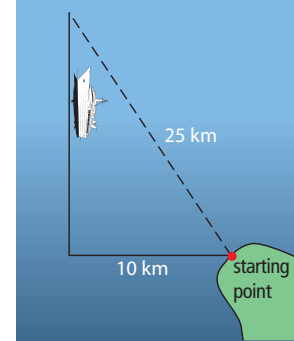
## Applying the Pythagorean Relationship

### Focus on...

After this lesson, you will be able to...

- apply the Pythagorean relationship to solve problems
- determine distances between objects

A ship leaves the Pacific coast of British Columbia and travels west for 10 km. Then, it turns and travels north. When the ship is 25 km from its starting point, how could you use the Pythagorean relationship to determine the distance the ship travelled north?

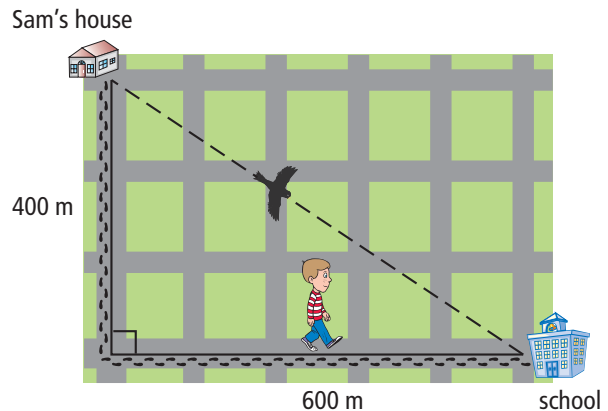


### Explore the Math

#### How can you determine a distance using the Pythagorean relationship?

The diagram shows Sam's trip to school.

1. a) Work with a partner to determine how far his house is from the school.
- b) Share your answer with your classmates. Is there more than one possible answer? Explain.



2. a) What do you think the expression "as the crow flies" means?
- b) How much farther does Sam travel than the crow? Show your method.

#### Reflect on Your Findings

3. Why is the path that the crow takes from Sam's house to the school difficult to measure directly?

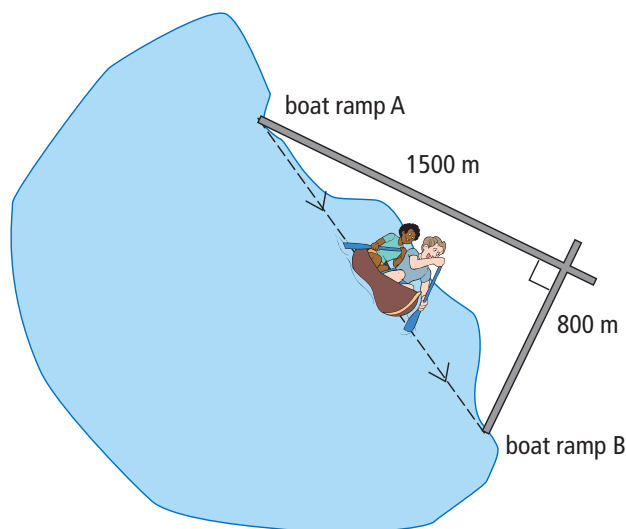
### Geography Link

North, south, east, and west are directions. On a compass, they are called the cardinal points.



### Example 1: Determine Distances With Right Triangles

- a) Anthony and Shalima are canoeing on a lake in Saskatchewan. There are two boat ramps on the lake. How far is it by canoe between the boat ramps?
- b) How much farther is it for someone to travel by road from ramp A to ramp B than to canoe between the two ramps?



### Solution

- a) The two roads leading from the boat ramps make the legs of a right triangle. The distance by canoe is the hypotenuse.

Let  $d$  represent the distance by canoe.

Use the Pythagorean relationship.

$$d^2 = 1500^2 + 800^2$$

$$d^2 = 2\,250\,000 + 640\,000$$

$$d^2 = 2\,890\,000$$

$$d = \sqrt{2\,890\,000}$$

$$d = 1700$$

The distance by canoe is 1700 m.

- b) Determine the total distance by road between the boat ramps.

$$1500 + 800 = 2300$$

The total distance by road is 2300 m.

Determine the difference between the two distances.

$$2300 - 1700 = 600$$

It is 600 m farther to travel by road than by canoe between the boat ramps.

### Strategies

Solve an Equation

### Strategies

What other method could you use to solve this problem?

### Show You Know

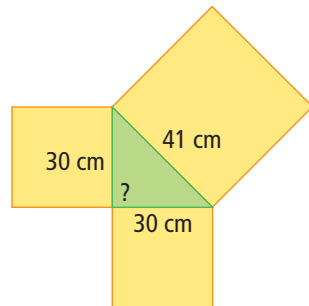
Refer to the opening paragraph and picture on page 106. A ship leaves the Pacific coast of British Columbia and travels west for 10 km. Then, it turns and travels north. If the boat is 25 km from its starting point, what distance did it travel north? Give your answer to the nearest tenth of a kilometre.

## Example 2: Verify a Right Angle Triangle

Danelle is trying to install a corner shelf in her bedroom. Since the shelf does not fit properly, she thinks the two walls in her bedroom do not meet at a right angle. She measures a length of 30 cm along the base of each wall away from the corner. Then, she measures the hypotenuse to be 41 cm. Do the walls meet at a right angle? Explain.



### Solution



#### Strategies

Draw a Diagram

#### Strategies

What other method could you use to solve this problem?

Use the Pythagorean relationship to determine whether the triangle is a right triangle.

Determine whether the sum of the areas of the two smaller squares equals the area of the large square.

$$\begin{aligned}\text{Left Side:} \\ 30^2 + 30^2 &= 900 + 900 \\ &= 1800\end{aligned}$$

The sum of the areas of the two smaller squares is  $1800 \text{ cm}^2$ .

$$\begin{aligned}\text{Right Side:} \\ 41^2 &= 1681 \\ \text{The area of the large square is} \\ &1681 \text{ cm}^2.\end{aligned}$$

$$1800 \text{ cm}^2 \neq 1681 \text{ cm}^2$$

The triangle is not a right triangle. The walls do not meet at a right angle.

### Show You Know

A construction company is digging a rectangular foundation with a width of 17 m and a length of 20 m. To check that a corner is a right angle, a worker measures the diagonal length, which is 26.25 m. Is the corner a right angle? Explain.

## Key Ideas

- The Pythagorean relationship can be used to determine distances that might be difficult or impossible to measure.

$$d^2 = 500^2 + 1200^2$$

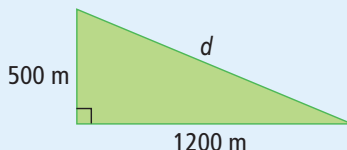
$$d^2 = 250\,000 + 1\,440\,000$$

$$d^2 = 1\,690\,000$$

$$d = \sqrt{1\,690\,000}$$

$$d = 1300$$

The hypotenuse is 1300 m.



- The Pythagorean relationship can be used to show if a triangle is a right triangle.

Left Side:

$$6^2 + 8^2 = 36 + 64$$

$$= 100$$

The sum of the areas of the two smaller squares is  $100 \text{ cm}^2$ .

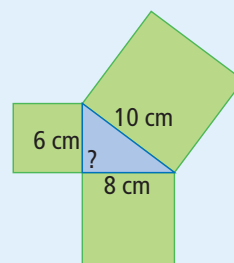
$$100 \text{ cm}^2 = 100 \text{ cm}^2$$

The triangle is a right triangle.

Right Side:

$$10^2 = 100$$

The area of the large square is  $100 \text{ cm}^2$ .



## Communicate the Ideas

- Use an example from real life to explain how you can apply the Pythagorean relationship to calculate distance.
- Ilana used the following method to determine whether the diagram shows a right triangle.

Left Side:

The large square is  $61 \text{ cm}$ .

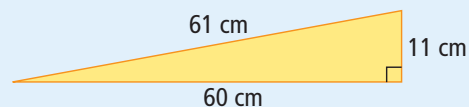
Right Side:

$$11 + 60 = 71$$

The two smaller squares are  $71 \text{ cm}$ .

$$61 \text{ cm} \neq 71 \text{ cm}$$

The triangle is not a right triangle.



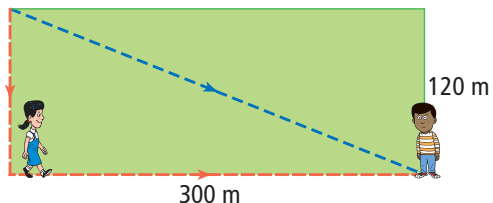
Is Ilana's method correct? If it is correct, explain how you know. If it is incorrect, explain the method Ilana should use.

# Check Your Understanding

## Practise

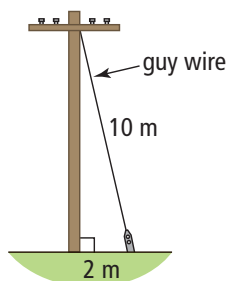
For help with #3 and #4, refer to Example 1 on page 107.

3. Walter walks across a rectangular field in a diagonal line. Maria walks around two sides of the field. They meet at the opposite corner.



- How far did Maria walk?
- How far did Walter walk?  
Express your answer to the nearest metre.
- Who walked farther? By how much?

4. Find the height of the pole where the guy wire is attached, to the nearest tenth of a metre.

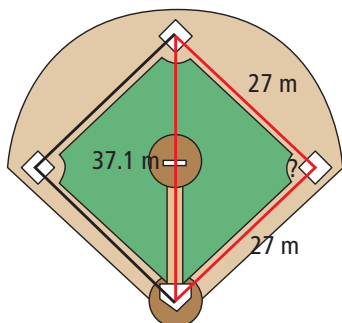


For help with #5 and #6, refer to Example 2 on page 108

5. Martin measured a rectangle and wrote:  
Width: 9 cm Length: 22 cm Diagonal: 23.8 cm

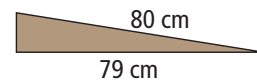
Could these measurements form a rectangle? Justify your answer.

6. You are asked to check the design plans for a baseball diamond. Is the triangle a right triangle? Explain.



## Apply

7. What is the height of the wheelchair ramp?  
Give your answer to the nearest tenth of a centimetre.



8. Shahriar knows that the size of a computer monitor is based on the length of the diagonal of the screen. He thinks that the diagonal is not as large as the ad says. Is he correct? Explain.

9. A checkerboard is made of 64 small squares that each have a dimension of 3 cm  $\times$  3 cm. The 64 small squares are arranged in eight rows of eight.
- What is the length of the diagonal of a small square? Give your answer to the nearest tenth of a centimetre.
  - What is the total length of the diagonal of the board? Give your answer to the nearest centimetre.

10. A gymnast requires a distance of 16 m for her tumbling routine. If the gymnast is competing on a 12 m  $\times$  12 m square mat, does she have enough room to do her routine safely? Explain your answer.

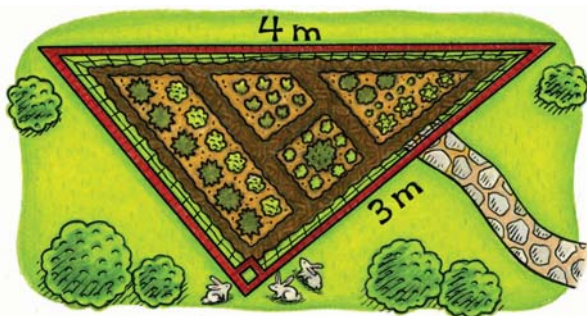




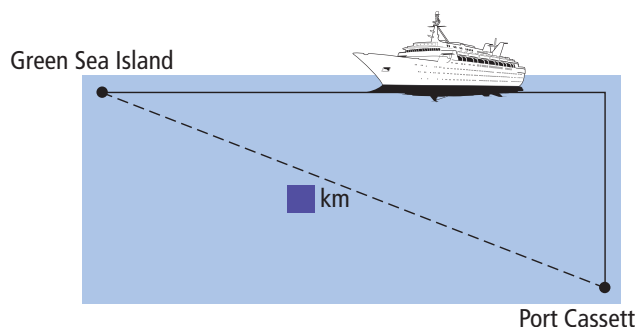
11. Johan has a 300-cm ladder that he leans up against a wall. The safety sticker on the side of the ladder shows that the bottom must be placed between 70 cm and 110 cm away from the wall. What are the minimum distance and maximum distance up the wall that the ladder can reach? Give your answers to the nearest tenth of a centimetre.

### Extend

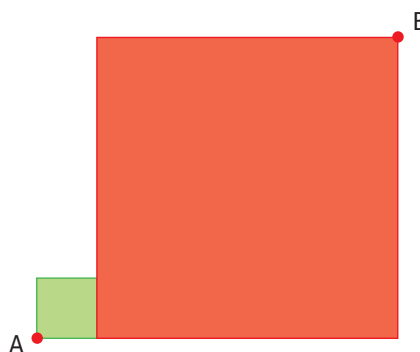
12. Sarah has a vegetable garden in the shape of a right triangle. She wants to put fencing all around it to keep the rabbits away.
- What total length of fencing does she need? Give your answer to the nearest hundredth of a metre.
  - If fencing costs \$2/m, what will be the total cost of the fencing?



13. A cruise ship travels from Port Cassett north at a speed of 34 km/h for 2.5 h. Then it turns  $90^\circ$  and travels west at 30 km/h for 7.3 h. When it reaches Green Sea Island, how far is the ship from Port Cassett? Express your answer to the nearest kilometre.



14. The red square has a perimeter of 40 mm and the green square has an area of  $4 \text{ mm}^2$ . What is the shortest distance between A and B? Give your answer to the nearest tenth of a millimetre.



### MATH LINK

The diagram shows the rough plans for a board game designed for a toy manufacturer. The board is composed of a square and four identical right triangles. Complete the plans by answering the following questions. Give your answers to the nearest tenth of a centimetre where appropriate.

- If the central square has an area of  $225 \text{ cm}^2$ , what is the perimeter of the game board? Show how you know.
- The game will be packaged in a box with a square base. Determine the minimum diagonal length of the base of the box.

