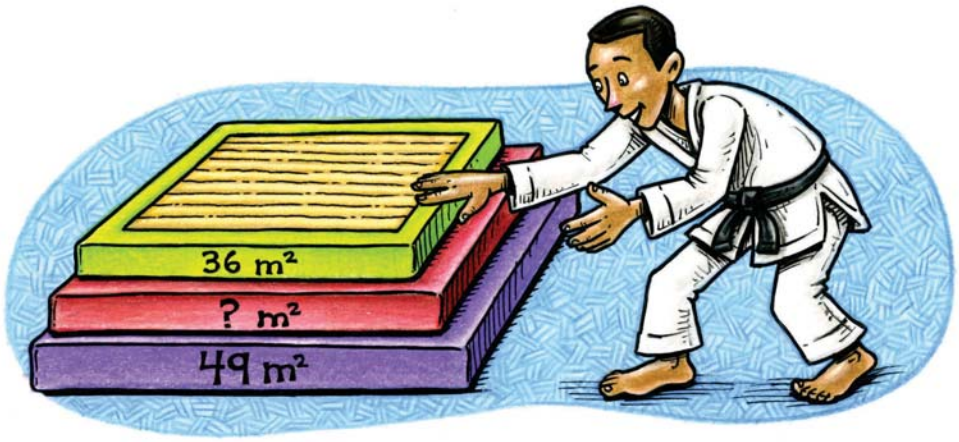
3.3

Estimating Square Roots

Focus on...

After this lesson, you will be able to...

- estimate the square root of a number that is not a perfect square
- identify a number with a square root that is between two given numbers



The picture shows three tatami mats that are used in judo. Can you think of a way to estimate the side length of the middle mat?

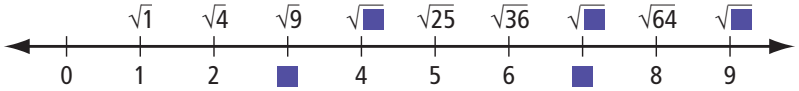
Explore the Math

How do you estimate a square root?

1. What is a reasonable estimate for the area of the middle mat in the picture?

2. What are the side lengths of the smallest and largest mats? Explain how you calculated these dimensions.

3. The number line below shows square roots of perfect squares. Copy the number line into your notebook. Complete the boxes.



4. Use the number line to estimate the side length for the middle mat. Give your answer to one decimal place.

Reflect on Your Findings

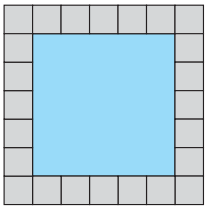
5. a) Compare your estimate of the side length of the middle mat with a classmate's.

   b) Using a calculator, determine the square root of your estimate in #1. Give your answer to the nearest tenth. Compare this approximation to your estimate for the side length.

   c) Explain how you can use perfect squares to estimate a square root.

Example 1: Estimate the Square Root of a Number

Felicity wants to know if a wading pool will fit in a small space in her yard. She must estimate the side length of the square wading pool, which has an area of 7 m^2.



a) What is a reasonable estimate for the side length of the pool? Use perfect squares to estimate. Give your answer to one decimal place.

b) Use a calculator to approximate the side length of the pool, to the nearest tenth of a metre. Compare your estimate in part a) with the calculator's approximate answer.

Solution

Strategies
Estimate and Check

a) The side length of the pool is the square root of 7.

The perfect squares on either side of 7 are 4 and 9.

Since 7 is closer to 9, the square root of 7 is closer to the square root of 9.

$\sqrt{9} = 3$

$\sqrt{7}$ will be a bit less than 3.

A reasonable estimate is 2.7 m.

b) Approximate the square root of 7.

C 7 √ 2.645751311

The answer to the nearest tenth of a metre is 2.6 m.

This answer is very close to the estimate of 2.7 m.

This value is an approximation. The decimal portion of the exact answer continues forever. The calculator can display only ten digits.
The square of the approximation shows that it is not an exact answer:
$2.645751311^2 = 6.999999999658218721$
≈ 7

Show You Know

For each of the following, use perfect squares to estimate the square root to one decimal place. Check your answer with a calculator.

a) $\sqrt{18}$ **b)** $\sqrt{23}$ **c)** $\sqrt{35}$

Example 2: Identify a Number With a Square Root Between Two Numbers

a) What is a whole number that has a square root between 6 and 7?

b) How many whole numbers can you find that have a square root between 6 and 7? Show your work.

Solution

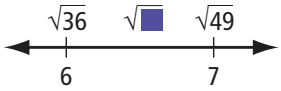
a) Determine the square of 6.

$6^2 = 36$

Determine the square of 7.

$7^2 = 49$

Draw a number line.



Find a value for ■ on the number line.

Choose any whole number between 36 and 49.

One possible whole number is 40.

$\sqrt{40}$ will have a value between 6 and 7.

Strategies
Estimate and Check

Check:

6.32455532 is between 6 and 7.

40 is a possible answer.

b) The possible answers are all of the whole numbers larger than 36 and smaller than 49:

37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48

There are 12 whole numbers that have square roots between 6 and 7.

Show You Know

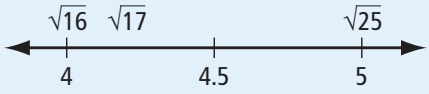
a) Identify a whole number with a square root between 8 and 9.

b) How many whole numbers can you find that have a square root between 8 and 9? Show your work.

Key Ideas

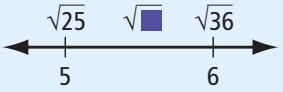
- To estimate the square root of a whole number that is not a perfect square,
  - locate the perfect squares on either side of the number
  - calculate the square roots of these two perfect squares
  - estimate based on the position between the two perfect squares

  For example, estimate the square root of 17:
  $\sqrt{17} \approx 4.1$



- To identify a whole number that has a square root between two given numbers,
  - determine the perfect squares of the two consecutive whole numbers
  - choose a whole number between the two perfect squares

  For example, identify a whole number that has a square root between 5 and 6:
  $5^2 = 25$ $6^2 = 36$
  $\sqrt{30}$ will have a value between 5 and 6.



- When using a calculator to find the square root of a natural number that is not a perfect square, the value shown on the calculator is only an approximation.

  C 8 √ 2.828427125

Communicate the Ideas

1. Explain how to estimate $\sqrt{28}$ to one decimal place without using a calculator. Compare your answer with a classmate's.

2. Find a whole number that has a square root between 3 and 4. Explain how you found it.

3. Jason is doing his math homework. He has to find the square root of 10. He presses √ **10** on his calculator and the screen displays 3.16227766. However, when 3.16227766 is multiplied by itself, the answer is not 10. Explain.

Check Your Understanding

Practise

For help with #4 to #5, refer to Example 1 on page 96.

4. Estimate the square root of each number, to one decimal place. Check with a calculator.

a) 72 **b)** 103 **c)** 55

5. Estimate each value, to one decimal place. Check your answer with a calculator.

a) $\sqrt{14}$ **b)** $\sqrt{86}$ **c)** $\sqrt{136}$

For help with #6 to #9, refer to Example 2 on page 97.

6. What is an example of a whole number that has a square root between 9 and 10?

7. Identify a whole number with a square root between 11 and 12.

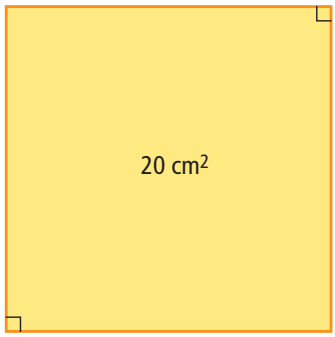
8. Identify all possible whole numbers with a square root larger than 2 and smaller than 3.

9. What are all possible whole numbers that have a square root between 4 and 5?

Apply

10. Kai uses an entire can of paint on a square backdrop for the school play. The label on the can states that one can covers 27 m^2 of wall surface. Estimate the backdrop's side length, to one decimal place.

11. The square has an area of 20 cm^2.



a) Use perfect squares to estimate the side length to one decimal place.

b) Check your answer using a ruler to measure the side of the square. Measure to the nearest tenth of a centimetre.

12. While shopping online, Ji Hun finds a square rug with an area of 11 m^2. He needs to know if it will fit in his 4 m × 5 m bedroom.

a) Estimate the side length of the rug, to one decimal place.

b) Check your estimate with a calculator.

c) Will the rug fit? Explain.

13. Stella is planning an outdoor wedding. She would like a square dance floor with an area of 115 m^2.

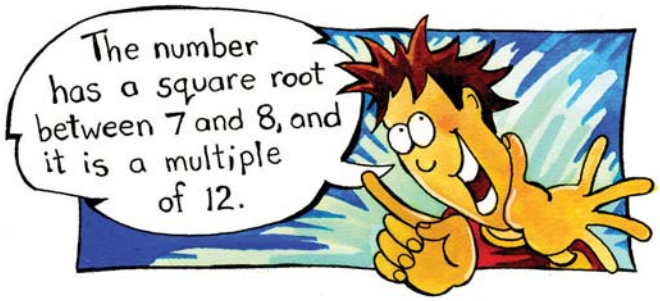
a) Determine the side length of the dance floor, to the nearest tenth of a metre.

b) Stella finds out that the dance floor will be made up of floorboards that each measure 1 m^2. What are the two side lengths the dance floor can have that are closest to what she wants?

c) What are the two square areas for the dance floor that Stella can choose from?

d) Which area will Stella choose? Explain.

14. Alex is thinking of a number.



a) What number could he be thinking of?

b) Is there more than one answer? Explain.

15. Order the following numbers from least to greatest: $7, \sqrt{46}, 5.8, \sqrt{27}, 6.3$.

16. A fitness centre will install a square hot tub in a 6 m × 6 m room. They want the tub to fill no more than 75% of the room's area.

a) What is the maximum area of the hot tub?

b) What dimensions, to a tenth of a metre, will the fitness centre order from the manufacturer? Explain.

17. Carmel wants to mount an 18 cm × 18 cm square picture on a square board that is four times the area of the picture.

a) What is the area of the picture?

b) What is the area of the board?

c) What are the dimensions of the board?

Extend

18. a) Evaluate $\sqrt{9}$.

b) Estimate the square root of your answer in part a), to one decimal place.

c) Use a calculator to check your estimate. Express your answer to the nearest hundredth.

d) How close is your estimate in part b) to your calculation in part c)?

19. Estimate $\sqrt{160\ 100}$. Explain how you determined your estimate.

20. What is the smallest natural number value for n if the solution for $\sqrt{56n}$ is also a natural number?

21. Determine two numbers that have a square root between 326 and 327, are divisible by 100, and are a multiple of 6.

MATH LINK

You have created a mini peg board game called Mind Buster. The square game board has a base area of 134 cm^2. You go to the store to get a box for storing the game. You find five boxes with the base dimensions shown.

a) Identify which boxes can store the game board. Explain.

b) Which box would you choose? Why?