3.2 Exploring the Pythagorean Relationship

Focus on...

After this lesson, you will be able to...

- ❑ model the Pythagorean relationship
- ❑ describe how the Pythagorean relationship applies to right triangles

Right triangles are found in art, construction, and many other objects. The sail for this sailboat is a right triangle. What makes this shape so special? You will explore some important properties of right triangles in this lesson.

Explore the Math

Materials

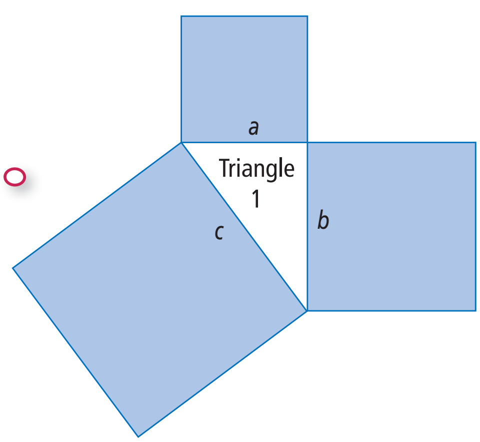
- centimetre grid paper
- scissors
- transparent tape
- protractor

What is a relationship that applies to right triangles?

1. From a piece of centimetre grid paper, cut out three squares with the following dimensions:
   6 cm × 6 cm 8 cm × 8 cm 10 cm × 10 cm

2. Arrange the squares to form Triangle 1 as shown. Tape the squares onto a sheet of paper. Label Triangle 1.

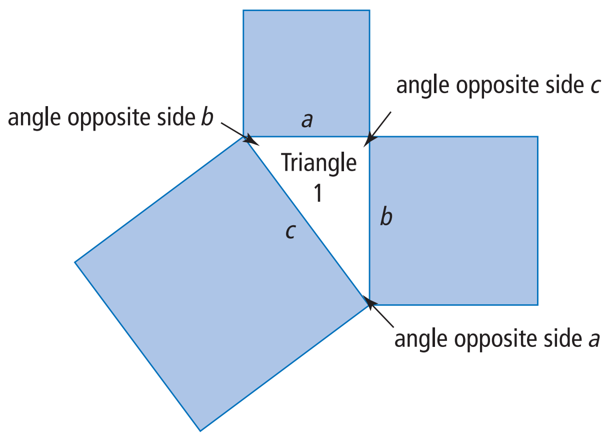
The length of side *a* is 6 cm, side *b* is 8 cm, and side *c* is 10 cm.



3. Copy the table below into your notebook.

	Side	Side Length (cm)	Angle Opposite the Side (°)	Area of Square (cm^2)	Right Triangle? (yes/no)
Triangle 1	a	6	37		
	b	8			
	c	10			
Triangle 2	a	5			
	b	7			
	c	10			
Triangle 3	a	5		25	
	b			144	
	c			169	

4. Measure the angle opposite each side of Triangle 1 with a protractor.



5. In your table, record the angle measures to the nearest degree.

6. Complete the rest of the table for Triangle 1.

7. Repeat the above steps for Triangles 2 and 3 in the table.

Reflect on Your Findings

8. a) Which triangles are right triangles? How do you know?

b) For each right triangle, write an addition statement showing the relationship between the areas of the three squares.

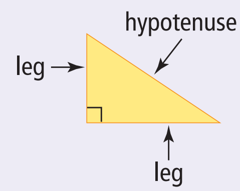
c) For each right triangle, describe in words the relationship between the side lengths of the triangle.

Literacy Link

Right Triangle

A *right triangle* has a right angle (90°). The right angle may be marked with a small square.

The two shorter sides that form the right angle are called the legs. The longest side is called the **hypotenuse**.



hypotenuse

- the longest side of a right triangle
- the side opposite the right angle

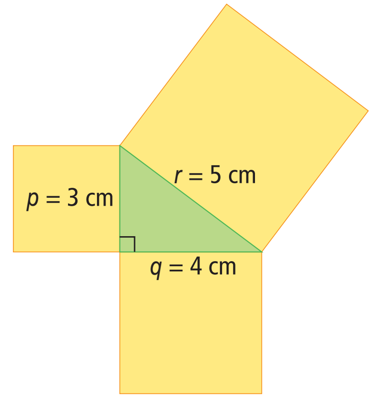
Example 1: Describe Relationships in Right Triangles

a) What is the area of each square?

b) Which side is the hypotenuse of the triangle?

c) Write an addition statement showing the relationship between the areas of the three squares.

d) Describe, using words and symbols, the relationship between the side lengths of the triangle.



Solution

a)

$p = 3$ cm	$q = 4$ cm	$r = 5$ cm
$A = 3^2$	$A = 4^2$	$A = 5^2$
$A = 9$	$A = 16$	$A = 25$
The area is 9 cm^2.	The area is 16 cm^2.	The area is 25 cm^2.

b) Side r is the hypotenuse.

This relationship is called the **Pythagorean relationship**.

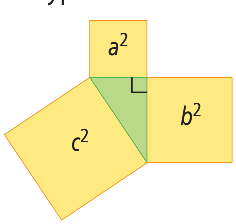
c) $9 + 16 = 25$

d) The sum of the areas of the squares attached to legs p and q equals the area of the square attached to hypotenuse r.

For a right triangle with legs p and q and hypotenuse r, $p^2 + q^2 = r^2$.

Pythagorean relationship

- the relationship between the lengths of the sides of a right triangle
- The sum of the areas of the squares attached to the legs of a right triangle equals the area of the square attached to the hypotenuse.



$a^2 + b^2 = c^2$

Show You Know

The sides of a right triangle are 9 cm, 12 cm, and 15 cm.

a) Sketch a picture of the triangle. Draw a square on each side of the triangle.

b) What is the area of each square?

c) Write an addition statement using the areas of the three squares.

Example 2: Identify a Right Triangle

A triangle has side lengths of 5 cm, 7 cm, and 9 cm.

a) What are the areas of the three squares that can be drawn on the sides of the triangle?

b) Is the triangle a right triangle? Explain your answer.

Web Link

To learn more about the Pythagorean relationship, go to www.mathlinks8.ca and follow the links.

Literacy Link

The symbol ≠ means "is not equal to."

Solution

a) $5 \times 5 = 25$
The area is 25 cm^2.

$7 \times 7 = 49$
The area is 49 cm^2.

$9 \times 9 = 81$
The area is 81 cm^2.

b) Calculate the sum of the areas of the two smaller squares.
$25 + 49 = 74$
The sum of the areas is 74 cm^2. The sum does not equal the area of the large square. 74 cm^2 $\neq$ 81 cm^2
The triangle is not a right triangle.

Show You Know

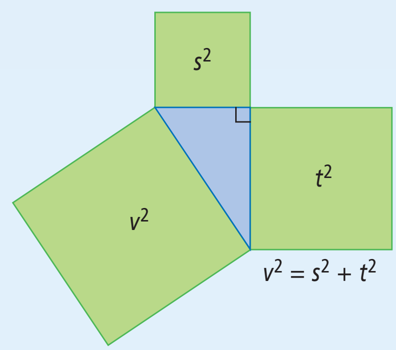
A triangle has side lengths of 12 cm, 16 cm, and 20 cm.

a) What are the areas of the three squares that can be drawn on the sides of the triangle?

b) Is the triangle a right triangle? Explain.

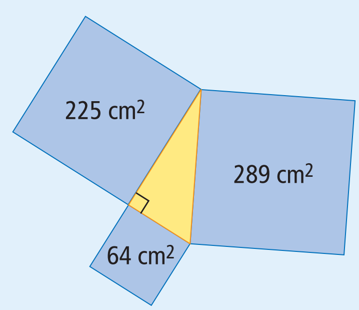
Key Ideas

- In a right triangle, the sum of the areas of the squares attached to the legs equals the area of the square attached to the hypotenuse.
- The Pythagorean relationship states that in a right triangle with sides s, t, and v, where side v is the hypotenuse, $v^2 = s^2 + t^2$.



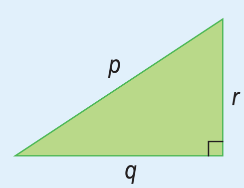
Communicate the Ideas

1. Describe, using words and symbols, the relationship among the areas of the three squares shown.



2. A triangle has side lengths of 7 cm, 11 cm, and 15 cm. Explain how you can determine whether or not it is a right triangle.

3. For the triangle shown, Kendra wrote the Pythagorean relationship as $r^2 = p^2 + q^2$. Is she correct? Explain.

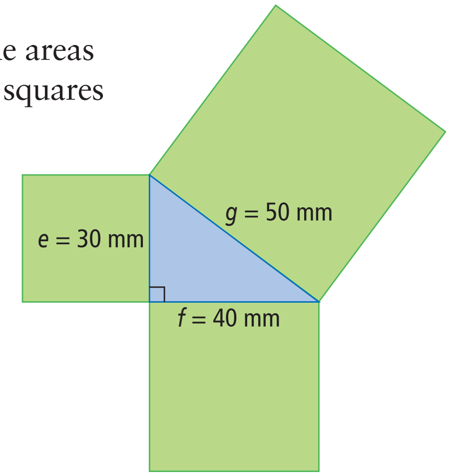


Check Your Understanding

Practise

For help with #4 to #7, refer to Example 1 on page 90.

4. What are the areas of the three squares shown?



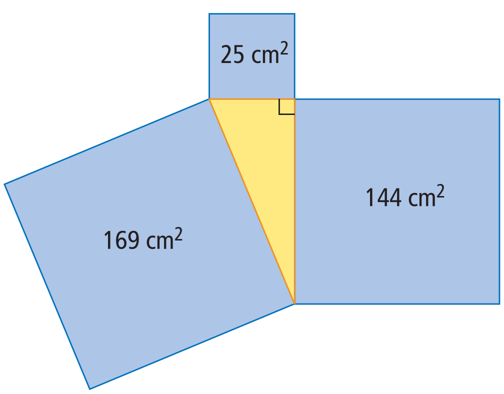
5. A right triangle has side lengths of 40 mm, 75 mm, and 85 mm.

a) Sketch the triangle. Draw a square on each side of the triangle.

b) What are the areas of the three squares?

c) Write an addition statement with the areas of the three squares.

6. a) Write an addition statement using the areas of these three squares.



b) What is the side length of each square?

c) Describe, using words and symbols, the relationship between the side lengths of each square.

7. The sides of a right triangle measure 9 cm, 12 cm, and 15 cm.

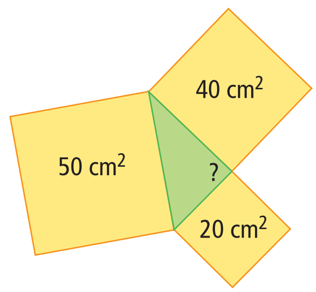
a) What is the area of each square attached to the three sides of the right triangle?

b) Write an addition statement showing the relationship between the areas of the three squares.

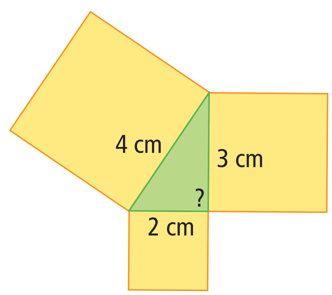
c) Describe, using words and symbols, the relationship between the side lengths of each square.

For help with #8 to #11, refer to Example 2 on pages 90–91.

8. Is the triangle shown a right triangle? Explain your reasoning.



9. a) Calculate the areas of the three squares.



b) Is this triangle a right triangle? Explain.

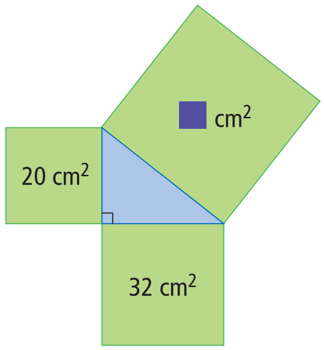
10. A triangle has side lengths of 120 mm, 160 mm, and 200 mm. Is the triangle a right triangle? Explain your reasoning.

11. The side lengths of a triangle are 5 cm, 6 cm, and 8 cm. Determine whether the triangle is a right triangle. Explain.

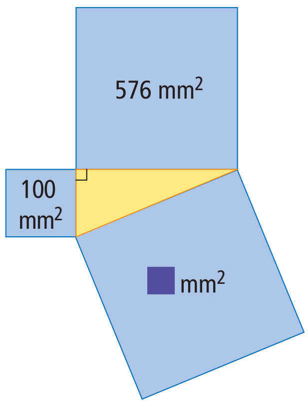
Apply

12. Use the Pythagorean relationship to find the unknown area of each square.

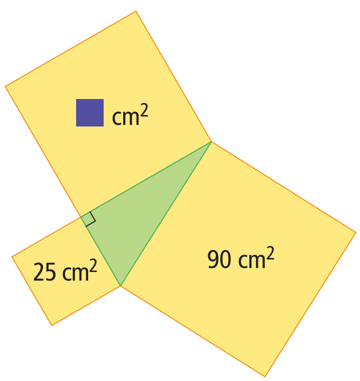
a)



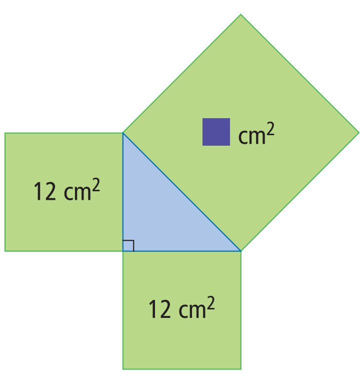
b)



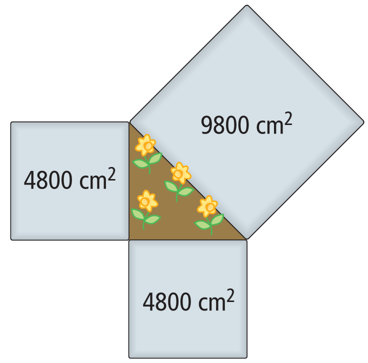
c)



d)



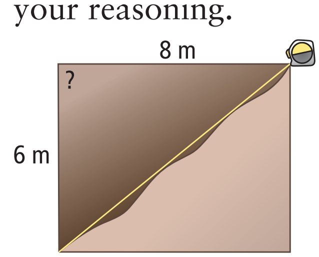
13. A small triangular flower bed has a square stepping stone at each of its sides. Is the flower bed in the shape of a right triangle? Explain your reasoning.



14. Show whether each triangle in the table is a right triangle.

Triangle	Side Lengths (cm)
A	9, 12, 15
B	7, 8, 11
C	7, 24, 25
D	16, 30, 34
E	10, 11, 14

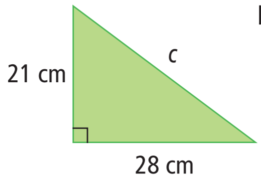
15. Construction workers have begun to dig a hole for a swimming pool. They want to check that the angle they have dug is 90°. They measure the diagonal as shown to be 9.5 m. Is the angle 90°? Explain your reasoning.



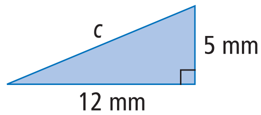
16. Baldeep is building a wooden box for storing coloured pencils. The box will have rectangular sides that are 12 cm wide and 20 cm long. Show how Baldeep can be sure the sides are rectangular, without using a protractor.

17. What is the area of the square that can be drawn on side *c* of each triangle?

a)

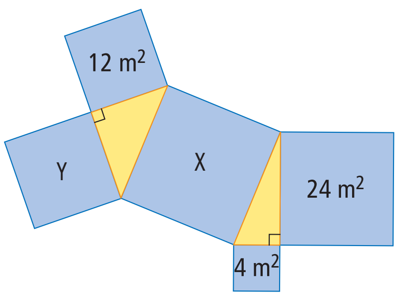


b)



Extend

18. The diagram is made of two right triangles and five squares.

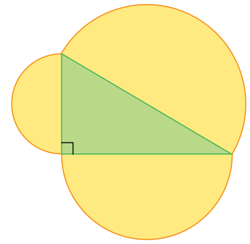


a) What is the area of square X?

b) What is the area of square Y?

19. A right triangle has a square attached to each side. Two of the squares have areas of 10 cm^2 and 15 cm^2. What are possible areas for the third square? Draw a sketch for each solution.

20. A right triangle has sides of 3 cm, 4 cm, and 5 cm. Attached to each side is a semi-circle instead of a square. Describe the relationship between the areas of the semi-circles.



Literacy Link

area of a circle $= \pi \times r^2$

21. An example of a Pythagorean triple is 3, 4, 5.

a) Multiply each number by 2. Show whether the resulting three numbers form a Pythagorean triple.

b) Multiply each number in the triple 3, 4, 5 by a natural number other than 2. Show whether the results form a Pythagorean triple.

c) Is there any natural number that does not make a Pythagorean triple when 3, 4, 5 are multiplied by it? Explain.

Did You Know?

A Pythagorean triple consists of three whole numbers that form the sides of a right triangle. For example, 3, 4, 5 make a Pythagorean triple because $3^2 + 4^2 = 5^2$.

MATH LINK

Identify the right triangle and three squares that complete this Pythagorean puzzle.