

# 2.2

## Rates

### Focus on...

After this lesson, you will be able to...

- express rates using words and symbols
- identify, describe, and record rates from real-life examples
- solve problems using rates



Trainers use technology to accurately and reliably monitor the heart rate of an equine competitor. Measuring the heart rate helps evaluate a horse's physical condition. The heart rate can be read at rest, during exercise, or during recovery after an event.

Heart rate is measured by counting the number of beats per minute. Note that a **rate** has two units. The units for heart rate are beats and minutes. Other common rates include growth rates and fuel efficiency rates. For example, a plant may grow 6 cm per month, and the fuel efficiency for a specific vehicle may be 6.8 L per 100 km.

What are some other rates you know about? What units are commonly used to measure these rates?

### rate

- compares two quantities measured in different units
- \$1.69 per 100 g or \$1.69/100 g is a rate for purchasing bulk food
- 72 beats per minute or 72 beats/min is a heart rate

### Materials

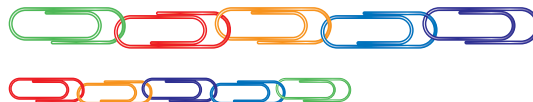
- standard paper clips
- jumbo paper clips

### Explore the Math

#### How can you determine a conversion rate?

Work with a partner. You will need a chain of standard paper clips and a chain of jumbo paper clips.

1. Use the paper clip chains to measure the lengths of six different objects in the classroom. Record your data.



### Did You Know?

Paper clips come in various sizes. In Canada, standard paper clips are about 33 mm in length. Jumbo clips are about 50 mm in length.

### unit rate

- a rate in which the second term is one
- for example, 20 km/h and 64 beats/min

### Did You Know?

Speed is a unit rate that compares the distance travelled to the time it takes.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

Speed is often measured in kilometres per hour or abbreviated as km/h.

2. What two units of measure are you using?
3. How can you use your data to determine a multiplier that describes the number of standard paper clips to one jumbo clip? This multiplier is called a conversion rate.

## Reflect on Your Findings

4. a) A conversion rate is sometimes called a **unit rate**. Explain why.  
b) Would the conversion rate for the number of jumbo clips for one standard clip be greater or less than one? Explain your thinking.  
c) Is the conversion rate between one jumbo clip and one standard clip always the same? Why or why not?

## Example 1: Determine Unit Rates

Ruby-throated hummingbirds and monarch butterflies travel similar paths across the Gulf of Mexico. The distance is just over 800 km. It takes the hummingbird 18.5 h and the monarch butterfly 41.6 h to cross the Gulf.



- a) Estimate the speed of the hummingbird and the butterfly.
- b) Calculate the speed of the hummingbird and the butterfly. Give each answer to the nearest hundredth.

**Solution**

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

	Hummingbird	Butterfly
a) Estimate speed.	$\frac{800 \text{ km}}{20 \text{ h}} = 40 \text{ km/h}$	$\frac{800 \text{ km}}{40 \text{ h}} = 20 \text{ km/h}$
b) Calculate speed.	$\frac{800 \text{ km}}{18.5 \text{ h}}$ $\text{C } 800 \div 18.5 \equiv 43.243243$ The speed is 43.24 km/h.	$\frac{800 \text{ km}}{41.6 \text{ h}}$ $\text{C } 800 \div 41.6 \equiv 19.230769$ The speed is 19.23 km/h.

**Strategies**  
Estimate and Check

A rate can be expressed as a fraction that includes the two different units. A rate cannot be expressed as a percent because a percent is a ratio that compares quantities expressed in the same units.

The speed of the hummingbird is 43.24 km/h and the speed of the monarch butterfly is 19.23 km/h, to the nearest hundredth. The estimates suggest that these answers are reasonable.

**Show You Know**

Determine the unit rate in each situation.

- a) Brandon runs 150 m in 25 s.
- b) Kira earns \$88 for working 8 h.
- c) Cat food costs \$9 for five cans.

**Example 2: Compare Prices Using Unit Rates**

Brett went to the grocery store to buy his favourite brand of orange juice. He found the following container sizes and prices. Which container of orange juice is the best buy?



### unit price

- a unit rate used when shopping
- often shown per 100 g or per 100 mL
- makes it easier for shoppers to compare costs of similar items



### Solution

Calculate the **unit price** of each container of orange juice and then compare.

414 mL for \$1.69

$$\begin{aligned}\text{Unit price} &= \frac{\text{cost}}{\text{volume}} \\ &= \frac{\$1.69}{414 \text{ mL}} \\ &= \$0.00408/\text{mL}\end{aligned}$$

$$\text{C } 1.69 \div 414 \equiv 0.0040821$$

The unit price is \$0.00408/mL or 0.408¢/mL.

946 mL for \$2.99

$$\begin{aligned}\text{Unit price} &= \frac{\text{cost}}{\text{volume}} \\ &= \frac{\$2.99}{946 \text{ mL}} \\ &= \$0.00316/\text{mL}\end{aligned}$$

$$\text{C } 2.99 \div 946 \equiv 0.0031607$$

The unit price is \$0.00316/mL or 0.316¢/mL.

1.89 L for \$5.49

To compare unit prices, the numbers must be in the same units.

$$\begin{aligned}\text{Unit price} &= \frac{\text{cost}}{\text{volume}} \\ &= \frac{\$5.49}{1890 \text{ mL}} \\ &= \$0.00290/\text{mL}\end{aligned}$$

$$\text{C } 5.49 \div 1890 \equiv 0.0029048$$

The unit price is \$0.00290/mL or 0.290¢/mL.

The unit price for the 1.89-L container is less than the unit prices of the other two containers. The best buy is the 1.89-L container for \$5.49.

\$1 = 100¢  
To convert dollars to cents, multiply by 100.

1 L = 1000 mL  
1.89 L = 1000 × 1.89  
= 1890 mL

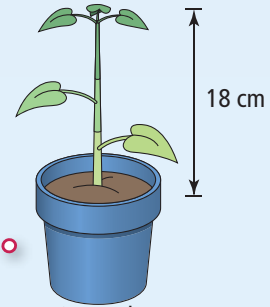
### Show You Know

At Ed's Grocery, one brand of salsa is sold in the following container sizes. Which container of salsa is the best buy? Show your work.



## Key Ideas

- A rate is a comparison of two quantities measured in different units.
  - A rate can be expressed as a fraction that includes the two different units. A rate cannot be expressed as a percent because a percent is a ratio that compares quantities expressed in the same units.



3 months

The growth rate compares height (in centimetres) and time (in months).

$$\text{Growth rate} = \frac{18 \text{ cm}}{3 \text{ months}}$$

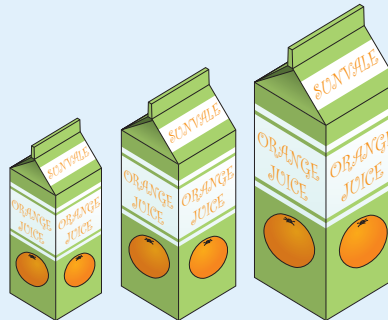
The plant grew 18 cm in 3 months.

- A unit rate is a rate in which the second term is one.

$$\frac{18 \text{ cm}}{3 \text{ months}} = \frac{6 \text{ cm}}{1 \text{ month}}$$

The plant grew at a rate of 6 cm/month.

- A unit price is a unit rate that makes it easier to compare the cost of similar items.



0.408¢/mL    0.316¢/mL    0.290¢/mL

0.290¢/mL < 0.316¢/mL < 0.408¢/mL

The largest container is the best buy.

## Communicate the Ideas

- Give an example of a ratio using words and numbers from the table.
  - What is a rate? Make up an example of a rate from the table.
  - Convert the rate in part b) to a unit rate.

Bear	Birth Mass (kg)	Mass After 60 Days (kg)
Black	0.3	6.5
Polar	0.7	7.4

- Two brands of canned dog food are on sale. Assume that the cans are the same size. Brand A costs \$13.60 for eight cans and Brand B costs \$8.75 for five cans. Explain how to find the unit price for Brands A and B. Explain how unit prices help you compare the cost of dog food.

3. a) Give two examples of rates that are common in every day life. Share your examples with a classmate.  
 b) What units measure each of the rates in part a)?  
 c) Explain why a rate cannot be expressed as a percent.

## Check Your Understanding

### Practise

For help with #4 to #6, refer to Example 1 on pages 56–57.

4. Determine the unit rate in each situation.
- An orca swims 110 km in 2 h.
  - A Canada goose flies 800 km in 12.5 h.
  - Cathy plants 45 daffodils in 30 min.
5. What is the unit rate in each?
- A blue whale eats 8 t of krill in 2 days.
  - The cruising speed of a blue whale allows it to travel 193 km in 10 h.
  - A bull moose bellows 15 times in  $2\frac{1}{2}$  h.
6. Gina earns \$78.00 for working 6 h. Asad makes \$192.50 after working 14 h. Determine each person's unit rate of pay. Who has a greater hourly rate of pay?

For help with #7 to #9, refer to Example 2 on pages 57–58.

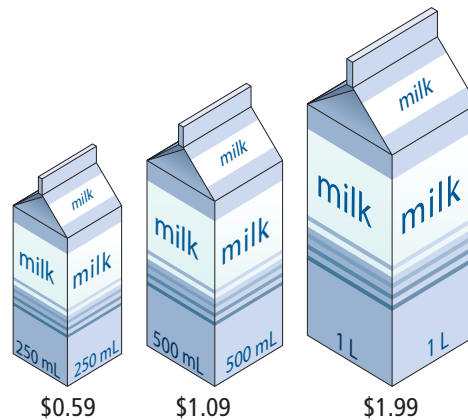
7. The table shows the price of different-sized packages of mixed nuts.

Nut Package	Mass	Price
1	300 g	\$2.19
2	500 g	\$3.09
3	700 g	\$4.83

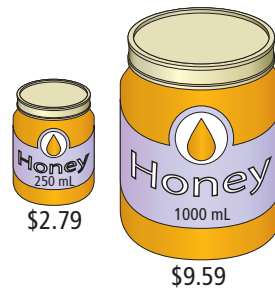
- a) What is the unit price per 100 g for each package?

- b) Which package is the best buy? Explain your choice.

8. Fraser is shopping for milk. It is available in three sizes.



- What is the unit price for each carton of milk?
  - What is the unit price per 100 mL for the 1-L carton?
  - Which carton of milk is the best buy? Explain why.
9. Mala is shopping for honey. Her favourite brand is available in two sizes.



- Estimate which is the better buy. Show your thinking.
- Determine the better buy. Show your work.

## Apply

10. Trevor rode his mountain bike 84 km in 3 h. Jillian rode 70 km in 2.5 h. Who is the faster cyclist? How do you know?
11. Shannon buys 12 granola bars for \$9.96.
- Determine the price per bar. Give your answer in dollars and cents.
  - Explain whether your answer in part a) is a ratio or a rate.
12. The rate at which glaciers melt is increasing globally. The Saskatchewan Glacier near Banff has receded 1.5 km in the last 75 years. The Peyto Glacier shown below receded 1320 m from 1923 to 1993. Which glacier had the greater annual rate of melting?



13. The table shows driving information for three drivers. Metric fuel consumption is measured in L/100 km, or litres per kilometre.

Driver	Distance (km)	Fuel Used (L)
Joe	400	28
Sarah	840	60
Martin	245	20

- What is the fuel consumption for Sarah's vehicle in litres per kilometre? Give your answer to four decimal places.
- How could you change the answer in part a) to express it in L/100 km?
- Which driver's vehicle had the lowest fuel consumption?

14. Conversion rates among currencies vary from day to day. The numbers in the table give the value in foreign currency of one Canadian dollar on one particular day.

Canadian	U.S.	Australian	European Union
1.00 dollar	0.8857 dollars	1.1527 dollars	0.6940 euros

- What was the value of \$600 Canadian in euros?
  - What was the value of \$375 Canadian in U.S. dollars?
  - What was the value of \$450 Canadian in Australian dollars?
15. Cindy Klassen from Winnipeg, Manitoba, won five speed skating medals at the 2006 Olympics. As of March 2006, she held the world record in the 1000 m, the 1500 m, and the 3000 m distances. Her times are shown in the table.

Time (min : s)	Distance (m)
1 : 13.11	1000
1 : 51.79	1500
3 : 53.34	3000



- Express each time in seconds.
- What was Cindy's speed in metres per second for her 1500 m record?
- How far does she skate in 10 s for the 3000 m distance?

## Extend

16. Twins, Daniel and Grace, take turns mowing the lawn. Last week Grace mowed the lawn in 45 min. This week Daniel mowed the lawn in 40 min.
- What is the average mowing rate per hour for each twin? Give each answer to the nearest hundredth.
  - What is the difference between Daniel's and Grace's mowing rates?
17. The time it takes a planet to make one revolution of its axis is a day on that planet. Consider each planet to be a sphere. So, if you are standing on the equator of a planet, you are travelling in a circle as the planet spins on its axis. Use the table to find the rotation rate in kilometres per hour for each planet.

Planet	Radius at Equator (km)	Length of Day (h)
Venus	6 051	2 808
Earth	6 378	24
Saturn	60 268	10 233

The formula relating the circumference,  $C$ , of a circle to its radius is  $C = 2 \times \pi \times r$ .

18. Chad went to the bank to get some U.S. dollars for a trip to the Grand Canyon. He paid \$500 Canadian and received \$441.15 U.S.
- What was the conversion rate for exchanging Canadian dollars to U.S. dollars? Give your answer to four decimal places. What does your answer represent?
  - How many U.S. dollars would Chad receive for \$700 Canadian at the rate in part a)?
  - Two days later, Chad returned to the bank and converted the \$441.15 U.S. back to Canadian dollars. He received only \$492.25 Canadian. What was the bank's conversion rate on that day for exchanging U.S. dollars to Canadian dollars? Give your answer to four decimal places.
  - How many U.S. dollars would Chad receive for \$700 Canadian at the rate in part c)?
19. Express 60 km/h in metres per second.

## MATH LINK

Kheer is a traditional rice pudding made in India and Pakistan. Pakistani kheer tends to be thicker than the Indian version. Look at the recipe for kheer. If the original recipe serves four people, calculate the quantity of each ingredient you need to serve 10 people. Use ratios and rates to support your reasoning.



### Kheer

#### Ingredients:

- 125 mL rice (basmati)
- 1 L milk
- 50 mL raisins
- 250 mL sugar
- 5 mL cardamom (or nutmeg)
- 50 mL almonds (slivered)

#### Method:

- Wash rice well.
- Boil milk and add rice. Simmer on low heat until rice is soft, stirring frequently to prevent sticking.
- When the rice is cooked and the mixture gets a semi-thick creamy consistency, add sugar and stir well.
- Remove from heat and add cardamom, slivered almonds, and raisins.
- Serve warm or chilled.