Two-Term and Three-Term Ratios
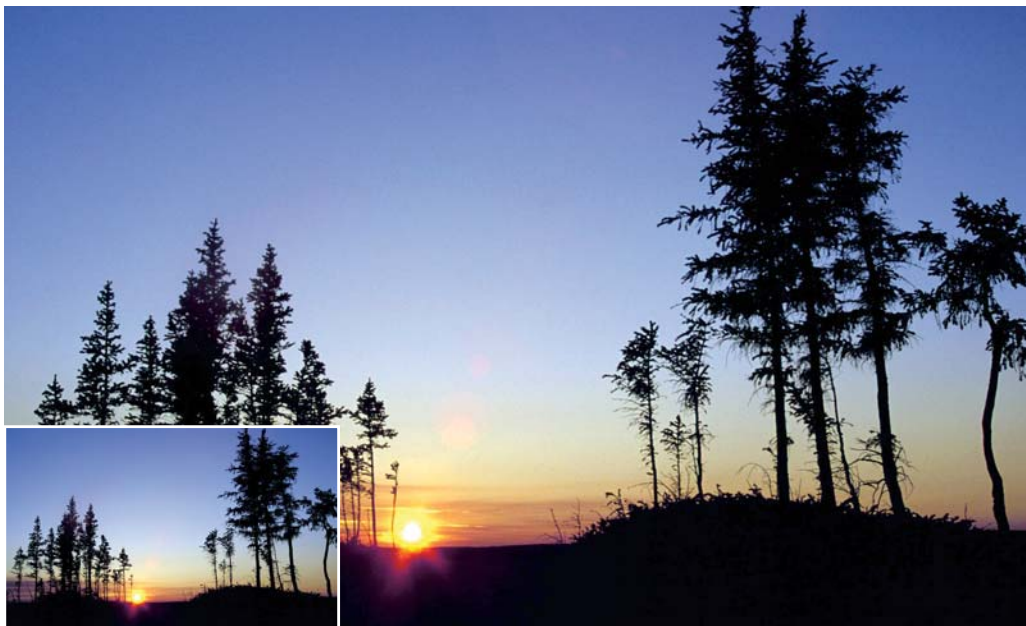
You use ratios to enlarge or reduce graphics and pictures. Discuss with a partner how you could use ratios to describe these two images.

Explore the Math

Materials
• ruler

How can you compare an enlargement to its original image?

Jason made the following enlargement of a prairie dog. Kate asked how he enlarged it. Jason could not remember. Use ratios to help him answer Kate's question.

1. a) How might you compare the original photo to its enlargement? Share your method with a classmate.

   b) What might you compare on the two photos to give you a ratio?

   c) How many measurements would you need to make?

   d) Why would it be important to measure exactly the same parts of each photo?

2. Suggest a multiplier that Jason would have used to enlarge the photo of the prairie dog. How did you arrive at your answer? Compare your answer with those of your classmates.

Reflect on Your Findings

3. a) If the ratio comparing the image to the original is greater than one, what does this tell you?

   b) If the ratio comparing the image to the original is less than one, what does this tell you?

Example 1: Represent Ratios

A bag contains 20 marbles.
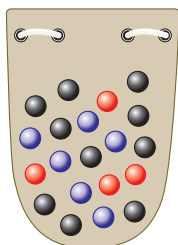
a) What is the two-term ratio of black to red marbles?

b) Compare the number of red marbles to the total number of marbles. Write the ratio as an equivalent fraction in lowest terms.

c) What marbles are represented by the ratio $6:10$?

d) Write the three-term ratio comparing the red, purple, and black marbles.
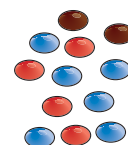
two-term ratio
- compares two quantities measured in the same units
- written as $a:b$ or a to b
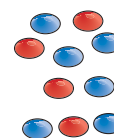
blue : red is $6:4$

three-term ratio
- compares three quantities measured in the same units
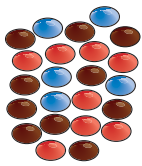- written as $a:b:c$ or a to b to c

blue : red : brown is $6:4:2$

Solution

a) *Method 1: Represent a Ratio Using a Drawing*

There are 10 black and 4 red marbles. The drawing shows the ratio.

This is a **part-to-part** ratio.

Method 2: Represent a Ratio Using Symbols or Words

The ratio of black marbles to red marbles is $10:4$ or 10 to 4.

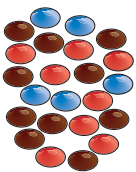**part-to-part ratio**

- compares different parts of a group to each other
- $10:8$ is the part-to-part ratio of brown to red beads.

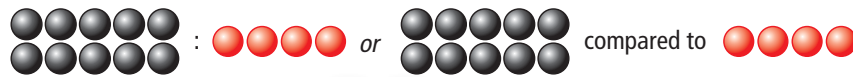**b)** There are 4 red marbles out of a total of 20 marbles.
The ratio can be expressed as $4:20$.
A **part-to-whole ratio** can be expressed as a fraction.

$$\frac{\text{red}}{\text{total}} \text{ is } \frac{4}{20}$$

Write the fraction as an equivalent fraction in lowest terms.

$$\frac{4}{20} \overset{\div 4}{\underset{\div 4}{=}} \frac{1}{5}$$

To express a fraction in lowest terms, divide the numerator and the denominator by the same number.

part-to-whole ratio

- compares one part of a group to the whole group
- $5:23$ is the part-to-whole ratio of blue to total number of beads.

c) There are 6 purple marbles and 10 black marbles.
purple : black is $6:10$

d) You can compare red, purple, and black marbles using a three-term ratio.

$$\text{red} : \text{purple} : \text{black} = 4:6:10$$
$$= 2:3:5$$

You can multiply or divide each term of a three-term ratio by the same number.

Show You Know

Use the design to help answer the following questions.

a) What is the ratio of red tiles to total titles? Express the answer three different ways.

b) What could the ratio $4:6$ represent?

c) What is the ratio of red to black to white tiles?

Example 2: Apply Ratios

Tamara has a recipe for fruit punch that calls for three cans of frozen orange juice concentrate, two cans of raspberry juice concentrate, and one can of lime juice concentrate. For each can of juice concentrate, the directions say to add three cans of water. All the cans are the same size. Tamara makes one recipe of fruit punch.
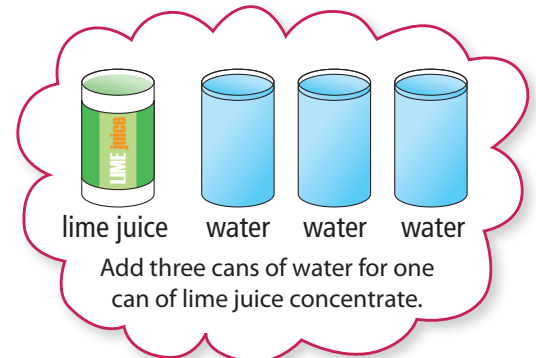
a) Copy and complete the following chart.

	Orange	Raspberry	Lime	Total
Juice Concentrate (cans)				
Water (cans)				
Total Punch (cans)				

b) What is the ratio of orange juice to lime juice concentrate? Express the ratio two different ways.

c) What is the ratio of lime to orange to raspberry juice concentrate?

d) What is the ratio of water to juice concentrate?

e) How many cans of punch does the recipe make?

f) What is the ratio of orange, raspberry, and lime juice concentrate to total punch? Express the ratio as a fraction, a decimal, and a percent.

Solution

a)

	Orange	Raspberry	Lime	Total
Juice Concentrate (cans)	3	2	1	6
Water (cans)	$3 \times 3 = 9$	$2 \times 3 = 6$	$1 \times 3 = 3$	18
Total Punch (cans)				24

lime juice   water   water   water

Add three cans of water for one can of lime juice concentrate.

b) three cans of orange juice and one can of lime juice concentrate
The ratio of orange to lime juice concentrate is $3:1$ or 3 to 1.

This is a part-to-part ratio.

c) one can of lime juice, three cans of orange juice, and two cans of raspberry juice concentrate
The ratio of lime to orange to raspberry juice concentrate is $1:3:2$.

d) 18 cans of water, 6 cans of juice concentrate
The ratio of water to juice concentrate is $18:6$ or $3:1$.

$\div 6$

$\dfrac{18}{6} = \dfrac{3}{1}$

$\div 6$

e) 18 cans of water + 6 cans of juice concentrate = 24 cans of punch
One recipe makes 24 cans of punch.

f) 6 cans of juice concentrate, 24 cans of punch

$$\frac{\text{juice concentrate}}{\text{punch}} \text{ is } \frac{6}{24} \xrightarrow{\div 6} = \frac{1}{4}$$

$$\frac{1}{4} = 0.25$$

$$0.25 = 25\%$$

> To convert the decimal to a percent, multiply by 100 and add the percent symbol.

The ratio of juice concentrate to punch is $\frac{1}{4}$, 0.25, or 25%.

Show You Know

A recipe for trail mix calls for three cups of mini pretzels, two cups of roasted soy chips, one cup of raisins, and one cup of sunflower seeds. You make two batches of trail mix.

a) What is the ratio of mini pretzels to raisins? Express the ratio two different ways.

b) What is the ratio of roasted soy chips to sunflower seeds?

c) How many cups of mix do two batches make?

d) What is the ratio of soy chips and sunflower seeds to total trail mix? Express the ratio as a fraction, a decimal, and a percent.

Key Ideas

- A part-to-part ratio compares different parts of a group.

  The ratio of red to black tiles is $6:3$ or 6 to 3.
  The ratio in lowest terms is $2:1$ or 2 to 1.

- A part-to-whole ratio compares one part of a group to the whole group.

  The ratio of red to total tiles is $6:12$ or 6 to 12. The ratio in lowest terms is $1:2$ or 1 to 2. One out of every two tiles is red.

  A part-to-whole ratio can be written as a fraction, a decimal, and a percent.

  The ratio of $\frac{\text{red}}{\text{total}}$ is $\frac{6}{12}$ or $\frac{1}{2}$, 0.5, 50%.

- A three-term ratio compares three quantities measured in the same units.

  The ratio of red to black to blue tiles can be written as 6 : 3 : 3 or 6 to 3 to 3. The ratio in lowest terms is 2 : 1 : 1 or 2 to 1 to 1.
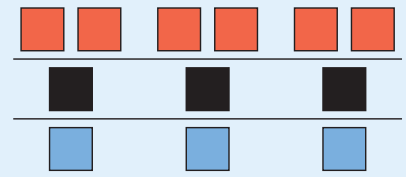
- A two-term ratio compares two quantities measured in the same units.

  The ratio of black to total tiles can be written as 3 : 12 or 3 to 12. The ratio in lowest terms is 1 : 4 or 1 to 4. One out of every four tiles is black.

Communicate the Ideas

1. Janine wants to write the ratio of oranges to apples. How does she know whether to write 3 : 4 or 4 : 3?

2. Your friend missed the class when ratios were introduced. Use an example and draw a diagram to explain the difference between a part-to-part ratio and a part-to-whole ratio.

3. Give two examples of how ratios are used in daily life. Share your ideas with a classmate.

4. The fraction $\frac{2}{5}$ can be interpreted as two parts out of a total of five parts. Use a diagram to show an example of this part-to-whole ratio.
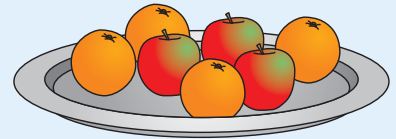
Check Your Understanding

Practise

For help with #5 to #8, refer to Example 1 on pages 47–48.

5. Write each ratio using ratio notation. Do not write the answers in lowest terms.
   a) $2 compared to $8.
   b) The width of the cover of this book compared to its length, in centimetres.
   c) In a class, 14 of 30 students are girls. What is the ratio of boys to girls to total students?
   d) Your age compared to that of a 28-year-old person.

6. Write each ratio in #5 as an equivalent ratio in lowest terms.

7. Write each ratio in fraction form. Do not write the answers in lowest terms.
   a) You spend $4 out of $10.
   b) A team won three games and lost six games. What is the ratio of games won to games played?
   c) A bag contains 12 red and 3 blue beads. Compare blue beads to total beads.
   d) A pond contains 27 guppies and 33 goldfish. What is the ratio of guppies to total fish?

8. Identify the missing number to make an equivalent fraction.

a) $\dfrac{1}{2} = \dfrac{\blacksquare}{8}$ b) $\dfrac{4}{5} = \dfrac{12}{\blacksquare}$

c) $\dfrac{2}{7} = \dfrac{\blacksquare}{21}$ d) $\dfrac{\blacksquare}{4} = \dfrac{3}{12}$

e) $\dfrac{21}{49} = \dfrac{3}{\blacksquare}$ f) $\dfrac{4}{\blacksquare} = \dfrac{12}{15}$

9. Use the data about wins and losses on school teams to answer the questions.

Sport	Wins	Losses
Hockey	9	6
Volleyball	10	5
Baseball	12	8

a) Which sports have equivalent win–loss ratios? Show how you know.

b) What is the ratio of wins to total games played for hockey? Give your answer as a fraction, a decimal, and a percent.

10. Tyler counted 20 cars in the school parking lot. Of these, 6 were red, 4 were green, and 1 was yellow.

a) Draw a diagram to represent the situation.

b) How many cars were not red, green, or yellow?

c) What is the ratio of yellow to green to red cars?

d) What is the ratio of red to total cars? Express the ratio as a fraction and a percent.

**11.** What tiles could be represented by each of the following ratios?

a) 1 to 5   b) $1:6:5$
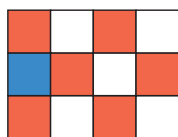
c) $\dfrac{1}{2}$ d) $\dfrac{11}{12}$

Apply

12. In a class of 32 students, there are 24 girls.

a) What is the boys to total students ratio? Express the ratio as a fraction and a percent.

b) What is the girls to boys ratio? Use ratio notation to express the ratio.

13. A soccer team played 28 games and won 4 out of every 7 games. There were no tied games.

a) How many games did they lose?

b) What was the team's win–loss ratio? Explain how you got your answer.

c) If this trend continues, how many losses would you expect the team to have once they have won 20 games?

14. Three eighths of the 96 adults in the McGregor clan are less than 150 cm tall.

a) Draw a diagram to represent the statement.

b) How many adults in the McGregor clan are less than 150 cm tall? Show your work using equivalent ratios.

c) How many adults are 150 cm or taller? Explain your thinking.

15. Diana and John are making three-cheese lasagna. The recipe calls for 100 g of Romano, 300 g of mozzarella, and 250 g of cottage cheese.

a) Write a ratio in lowest terms to compare the amounts of the three cheeses. State the order of the cheeses.

b) What amounts of Romano and cottage cheese do you need to make lasagna that contains 900 g of mozzarella cheese? Hint: Use equivalent ratios to help you.

16. Heather used a copier to make the following 50% reduction.

original   reduction
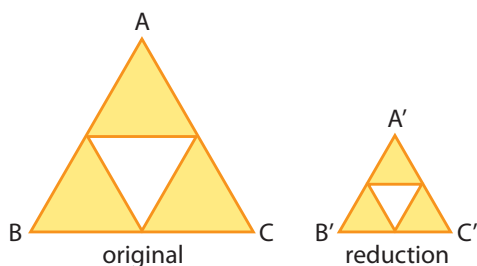
a) Write the ratio of the length of A′B′ compared to the length of AB.

> **Literacy Link**
>
> A′ is read as "A prime." A′ labels the point in the reduction that corresponds to point A.

b) Write the ratio of the length of A′C′ compared to the length of AC.

c) Use your knowledge of ratio and percent to explain the meaning of a 50% reduction.

17. There are 48 passengers on a transit bus. At the next stop, 16 passengers got off and 12 others got on the bus.

a) What is the ratio of the passengers who got off the bus compared to the original number on the bus? Show the ratio in lowest terms.

b) What is the ratio of the passengers who got on the bus at the stop compared to the new total then on the bus? Write your answer as a fraction, a decimal, and a percent.

18. The ratio of the width to the length of the Canadian flag is 1:2.

a) The flag on the cover of an atlas is 12 cm wide. How long is it?

b) A large flag outside a Calgary school is 3 m long. What is its width?

WWW Web Link

To practise solving ratio problems, go to www.mathlinks8.ca and follow the links.

19. The table gives the lengths of some rivers in Western Canada.

River	Length (km)
Churchill	1608
Fraser	1368
MacKenzie	1800
North Saskatchewan	1392
Thelon	904

a) Write a ratio in fractional form to compare the length of the Churchill River and the Mackenzie River. Express the ratio as a fraction in lowest terms.

b) Write a ratio in decimal form to compare the length of the North Saskatchewan River and the Fraser River. Express the ratio as a decimal correct to the nearest hundredth.

c) The calculator keying sequence that compares the length of the Thelon River and the Churchill River is

[C] 904 [÷] 1608 [=] 0.5621890547

Write the decimal to the nearest hundredth. What comparison statement can you make about the length of the two rivers?

20. A 30-kg bag of fertilizer is labelled 15–20–10. This means that it contains 15% nitrogen, 20% phosphorus, and 10% potassium by weight. How many kilograms of nitrogen, phosphorus, and potassium are in the bag?

> **Did You Know?**
>
> Fertilizer ratios indicate the percent of nitrogen, phosphorus, and potassium. This bag of fertilizer contains 15% nitrogen, 20% phosphorus, and 10% potassium by weight. The remaining 55% is made up of other micronutrients and filler.
>
>

Extend

21. A golden rectangle has a length to width ratio called the golden ratio, which is approximately 1.62.

a) Which of the following dimensions of rectangles are examples of golden rectangles?
- 24 m × 38.9 m
- 52 cm × 120.5 cm
- 348 mm × 565 mm

b) If the width of a golden rectangle is 6.4 m, what is its length? Give your answer to the nearest tenth of a metre.

Did You Know?

The golden rectangle is used often in art and architecture. For example, the front of the Parthenon, a temple in Athens, Greece fits into a golden rectangle.

22. The side view of a ramp is shown.
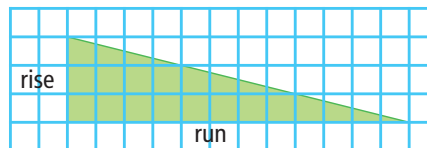
a) Express the ratio of rise to run in lowest terms. This ratio describes the slope of the ramp.

b) Express the slope ratio as a fraction, a decimal, and a percent.

c) Predict what effect each of the following would have on the slope of the ramp:
- increasing the rise
- decreasing the rise
- increasing the run
- decreasing the run

Did You Know?

The ratio $\frac{rise}{run}$ is called *slope*. Slope describes the steepness of roads, ramps, and ski runs.

WWW Web Link

For more information about the golden ratio in nature, architecture, art, poetry, and music, go to www.mathlinks8.ca and follow the links.

MATH LINK

Plan an invitation for your international meal. Create a logo as part of the front of the invitation. A logo is an identifying symbol used in advertising.

a) Design your logo using colours or measurements to show each of the following ratios

4:3   2:3:4

For example, if you use a rectangle in your logo, you could show that the length to width ratio is 4:3.

b) Draw the logo on a 36 cm² section of centimetre grid paper.

c) Identify the ratios used in your logo.