12.3

Constructing Tessellations Using Rotations

Focus on...

After this lesson, you will be able to...

- identify how rotations can be used to create a tessellation
- create tessellating patterns using two or more polygons

Did You Know?

Professor Ronald Resch of the University of Utah built the world's largest pysanka from 3500 pieces of aluminum. It is located in Vegreville, Alberta; weighs 2300 kg; is 9.4 m high, 7 m long, and 5.5 m wide; and turns in the wind like a weather vane!

Pysanky is the ancient Eastern European art of egg decorating. The Ukrainian version of pysanky is the most well known. The name comes from the verb *to write*, because artists use a stylus to write with wax on the eggshell. Can you see how rotations are used to make the patterns on these eggs?

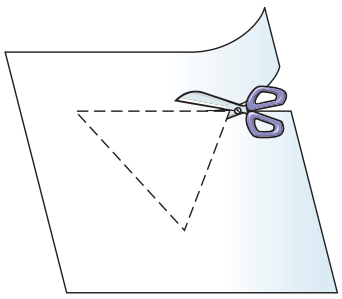
Materials

- tracing paper
- scissors
- glue stick
- tape
- cardboard or construction paper
- coloured pencils

Explore the Math

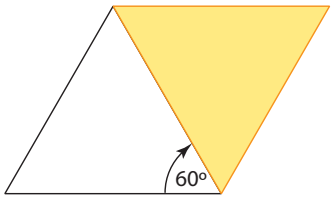
How can you create tessellations using rotations?

1. Draw an equilateral triangle with side lengths of 4 to 5 cm on a piece of paper. Cut out the triangle and glue it to a sheet of cardboard or construction paper to create a tile.



2. Trace around your tile on a piece of paper.

3. Rotate the tile 60° about one vertex until the edge of the tile falls along the edge of the previous tracing as shown. Trace around the tile again.



4. Repeat #3 until a full turn has been made.
   a) What shape did you create?
   b) How many times did you have to rotate the tile to create this shape?

5. Add colour and designs to the tessellation to make a piece of art.

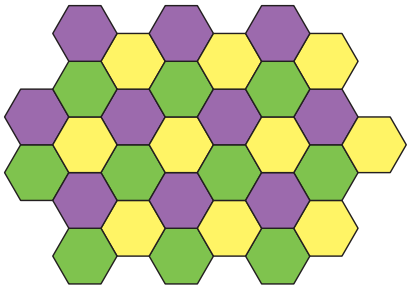
6. How could you continue to use rotations to make a larger tessellation?

Reflect on Your Findings

7. a) Describe how to use rotating polygons to create tessellations.
   b) What types of polygons can be used to make tessellations based on rotations? Explain.

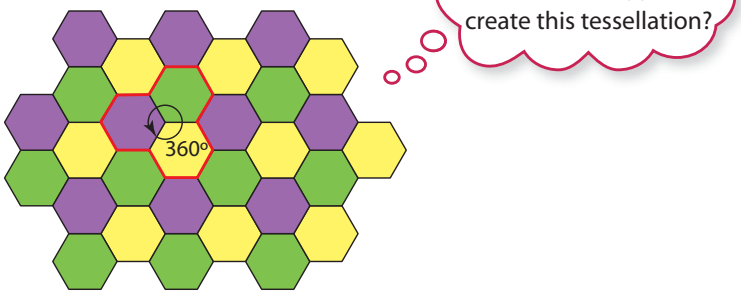
Example: Identify the Transformation

What polygons and what transformation could be used to create this tessellation?



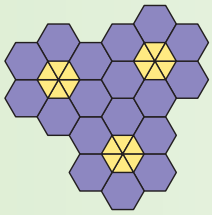
Solution

The tessellating tile is made up of a regular hexagon that has been rotated three times to make a complete turn. The three hexagons forming this tile can be translated horizontally and diagonally to enlarge the tessellation.



Show You Know

What polygons and transformations could be used to create this tessellation? Explain how you know.



Key Ideas

- Tessellations can be made with two or more polygons as long as the interior angles where the polygons meet total exactly 360°.
- Rotations can be used to create tessellations.

Communicate the Ideas

1. When creating a tessellation using rotations, why is it important for the sum of the angle measures at the point of rotation to equal 360°? Explain.

2. Describe to a partner how to use rotating polygons to create tessellations.

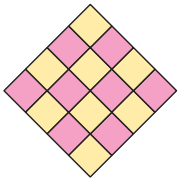
Check Your Understanding

Practise

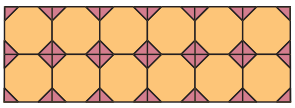
For help with #3 and #4, refer to the Example on page 458.

3. Identify the polygons used to create each tessellating tile.

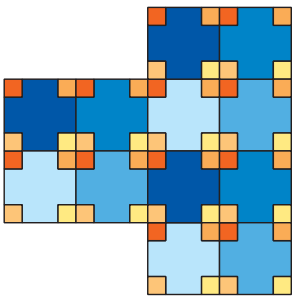
a)



b)



c)



4. What transformations could be used to create each tessellation in #3?

Apply

5. Examine the piece of stained glass.

a) Describe the transformation(s) used to make this pattern.

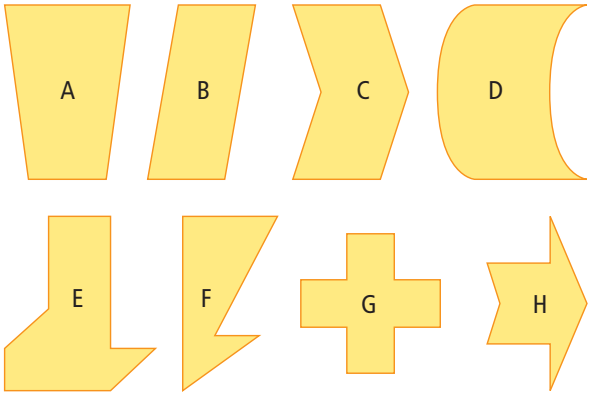
b) If you were using this pattern to tile the plane, what modifications would you have to make?

6. Design your own stained-glass window on grid paper. Describe the steps you followed to create the pattern.

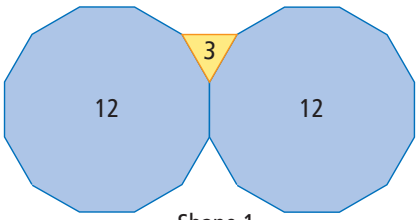
7. Create a tessellation using two different regular polygons and rotations.

Extend

8. Which of the following shapes tessellate? Explain how you know a shape will or will not tessellate.



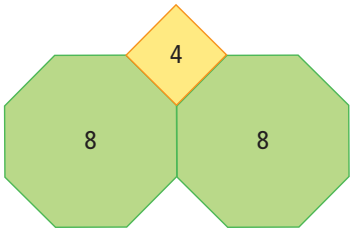
9. The diagram shows one arrangement of three or more polygons that can be used to create tessellations using rotations. One triangle and two dodecagons can be used because the angles at each vertex total 360° where they join. This is represented as (3, 12, 12). The table shows the features of this tessellation, for Shape 1.



Shape 1

Tessellations Involving Three Regular Polygons	Shape 1	Shape 2	Shape 3	Shape 4
Triangle (60°)	1			
Square (90°)	0			
Pentagon (108°)	0			
Hexagon (120°)	0			
Octagon (135°)	0			
Dodecagon (150°)	2			
Number of Sides	(3, 12, 12)			
Sum of Angles	60 + 2(150) = 360°			

a) Copy the table into your notebook. Complete the table for Shape 2 for the diagram shown.



Shape 2

b) Complete the table for Shapes 3 and 4, using different combinations of three or more regular polygons that total 360°.

c) Create construction paper or cardboard cutouts of the regular polygons from part b). Try to tessellate the plane using the combinations that you believe will work.

MATH LINK

Create your own pysanka design based on tessellating one or more polygons. Use at least one rotation in your design. Trace your design on grid paper, and colour it. Make sure it is the correct size to fit on an egg. If you have time, decorate an egg with your pysanka design.

WWW Web Link

To see examples of pysankas, go to www.mathlinks8.ca and follow the links.