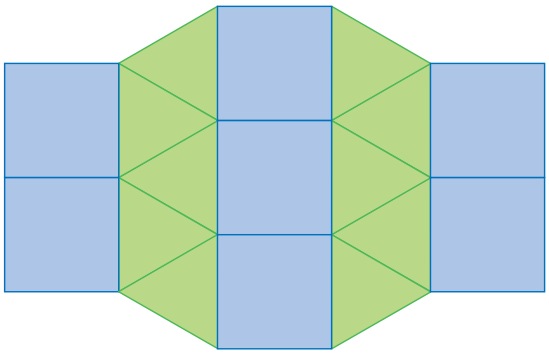
12.2

Constructing Tessellations Using Translations and Reflections



Focus on...

After this lesson, you will be able to...

- ❑ identify how translations and reflections can be used to create a tessellation
- ❑ create tessellating patterns using two or more polygons

In section 12.1 you created simple tessellating patterns using regular and irregular polygons. Tessellations can also be made by combining regular or irregular polygons and then transforming them. Do you recognize the polygons used in this tessellation? What **transformations** were used to create the pattern?

transformation

- a change in a figure that results in a different position or orientation

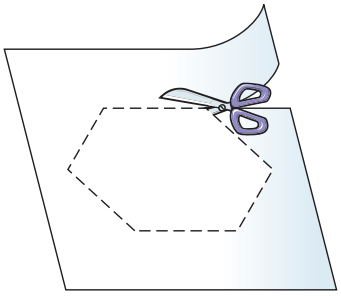
Explore the Math

How can you create a tessellation using transformations?

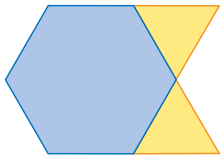
Materials

- set of pattern blocks, or cardboard cutouts of pattern block shapes
- ruler
- scissors
- glue stick
- tape
- cardboard or construction paper

1. Draw a regular hexagon on a piece of paper using a pattern block or cardboard cutout. Cut out the hexagon and glue it to a sheet of cardboard or construction paper.

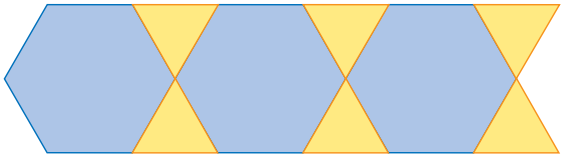


2. Draw two equilateral triangles on a piece of paper using a pattern block or cardboard cutout. Make sure that the side lengths of the triangles are the same as the side lengths of the hexagon. Cut out the triangles and glue them to a sheet of cardboard or construction paper so that they are attached to the sides of the hexagon as shown.

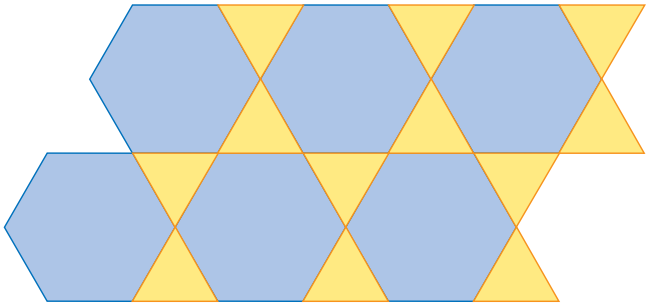


3. Cut out the combined shape. Trace the shape on a new sheet of paper.

4. Translate the shape so that the hexagon fits into the space formed by the two triangles. Trace around the translated shape and repeat two more times. What other ways can you translate the shape?



5. Translate the combined piece vertically and horizontally so that the base of the hexagon is now at the top of one of the triangles.



Reflect on Your Findings

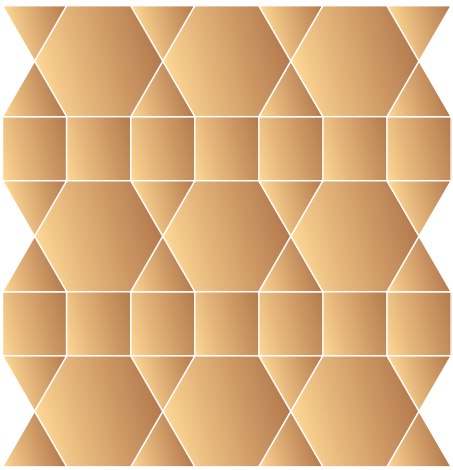
6. a) Describe how to use translations to create tessellations.

b) What other transformations could you use to get the same pattern as in #5? Explain the difference.

Example: Identify the Transformation

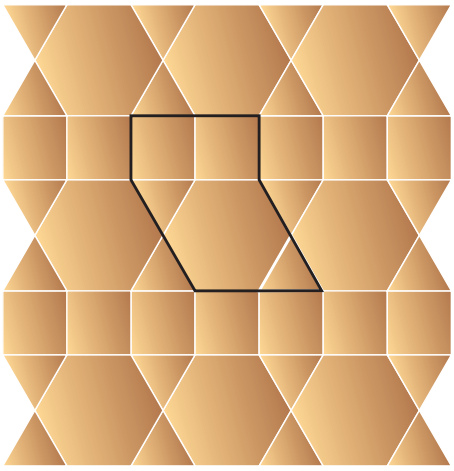
a) What polygons and what transformations are used to create this tessellation?

b) Does the area of the tessellating tile change during the tessellation?



Solution

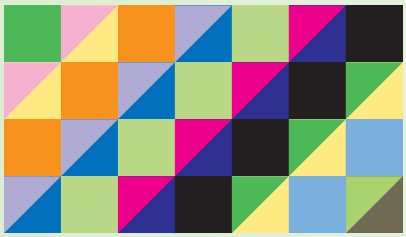
a) The tessellation is made from a tessellating tile consisting of a hexagon with two squares and two equilateral triangles. The tessellating tile is then translated vertically and horizontally. This tessellation is created using translations.



b) The area of the tessellating tile remains the same throughout the tessellation. There are no gaps or overlapping pieces.

Show You Know

What transformation was used to create this tessellation? Explain your reasoning.



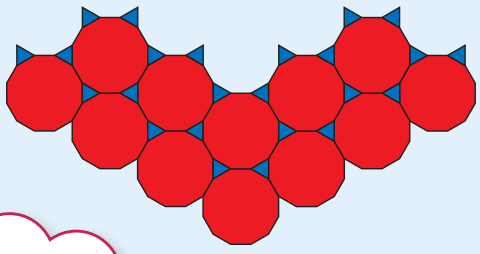
Key Ideas

- Tessellations can be made with two or more polygons as long as the interior angles where the vertices of the polygons meet total exactly 360°.
- Two types of transformations commonly used to create tessellations are
  - translations
  - reflections
- The area of the tessellating tile remains the same after it has been transformed to create a tessellation.

Communicate the Ideas

1. Brian missed today's class. How would you explain to him why some tessellating patterns made using translations could also be made using reflections?

2. Ashley and Vijay are trying to figure out how this tessellation was made. Whose answer is correct? Explain.



Ashley says:

The tessellation is based on reflecting the blue triangles across the red dodecagon.

Vijay says:

The tessellation is based on translating the red dodecagon with 2 blue triangles.

Literacy Link

A dodecagon is a 12-sided polygon.

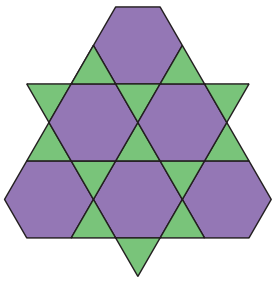
Check Your Understanding

Practise

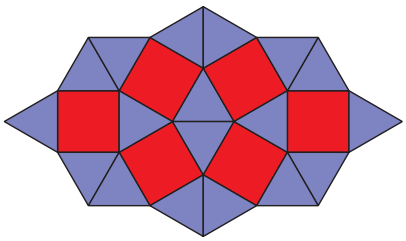
For help with #3 and #4, refer to the Example on pages 453–454.

3. Identify the two regular polygons used to create each tessellation.

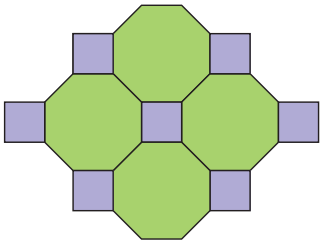
a)



b)



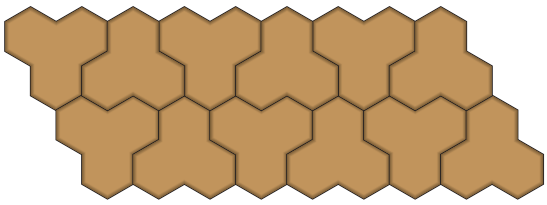
c)



4. What type of transformation could be used to create each tessellation in #3?

Apply

5. The diagram shows a garden path made from irregular 12-sided bricks.



a) Explain why the 12-sided brick tessellates the plane.

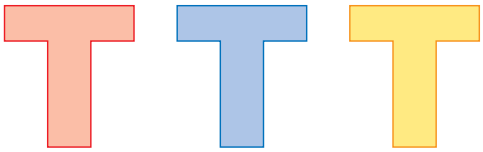
b) Use grid paper to design an irregular ten-sided brick that could be used to make a path.

c) Explain why your ten-sided brick tessellates the plane.

d) Use grid paper to design an irregular six-sided brick that could be used to tessellate the plane.

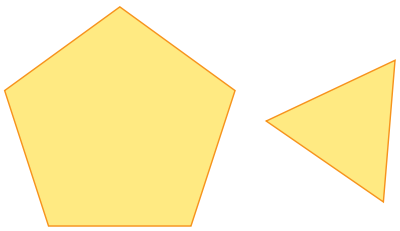
e) Explain why your six-sided brick tessellates the plane.

6. Simon is designing a wallpaper pattern that tessellates. He chooses to use the letter "T" as the basis of his pattern. Create two tessellations using the three coloured letters shown.



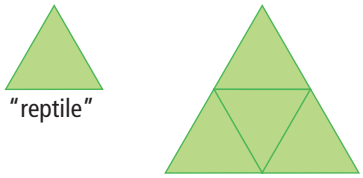
7. Priya is designing a kitchen tile that uses two different regular polygons. She then uses two different translations to create a tessellation. Use grid paper to design a tile that Priya could use. Show how it tiles the plane.

8. Barbara wants to make a quilt using the two polygons shown. Will she be able to create a tessellating pattern using these shapes? Explain.

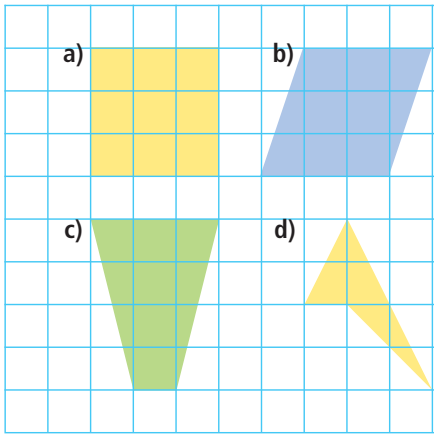


Extend

9. An equilateral triangle is called a reptile (an abbreviation for "repeating tile") because four equilateral triangles can be arranged to form a larger equilateral triangle.



Which of these figures are reptiles? Use grid paper to draw the larger figure for each reptile.



MATH LINK

Many quilt designs are made using tessellating shapes. This quilt uses fabric cut into triangles that are sewn together to form squares. The squares are then translated vertically and horizontally.

Design your own quilt square using one or more regular tessellating polygons. Create an interesting design based on patterns or colours.