12.1

Exploring Tessellations With Regular and Irregular Polygons

Focus on...

After this lesson, you will be able to...

- identify regular and irregular polygons that can be used to create tessellations
- describe why certain regular and irregular polygons can be used to tessellate the plane
- create simple tessellating patterns using polygons

Mosaics are often made of repeating patterns of tiles. What patterns do you see in the design?

Many mosaic tile designs are made from shapes that cover the area, or the plane, without overlapping or leaving gaps. These patterns are called **tiling patterns** or **tessellations**. Covering the plane in this way is called **tiling the plane**.

tiling pattern

- a pattern that covers an area or plane without overlapping or leaving gaps
- also called a **tessellation**

tiling the plane

- using repeated congruent shapes to cover an area without leaving gaps or overlapping
- also called tessellating the plane

Explore the Math

Which shapes can you use to tile or tessellate the plane?

1. Copy the following table into your notebook.

Shape	Regular or Irregular Polygon?	Measure of Each Interior Angle	*Prediction:* Will the shape tile the plane?	*Result:* Does the shape tile the plane?
Equilateral triangle				
Isosceles triangle				
Square				
Regular pentagon				
Regular hexagon				
Regular octagon				
Irregular quadrilateral				
Irregular pentagon				
Irregular hexagon				

Materials

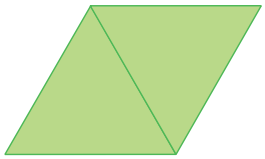
- set of pattern blocks, or cardboard cutouts of pattern block shapes
- protractor
- cardboard cutouts of an isosceles triangle, a regular pentagon, and a regular octagon
- cardboard
- scissors
- ruler

2. a) Select an equilateral triangle block. Is this a regular or irregular polygon? Record your answer in the table.

b) Measure each interior angle and record your measurements in the table.

c) Predict whether the shape will tile the plane. Record your prediction in the table.

3. Trace the outline of the equilateral triangle. Move the triangle to a new position, so that the two triangles share a common side. Trace the outline of the triangle again. Continue to see if the shape tiles the plane. Record your conclusion in the table.



4. Use the same method to find out if the isosceles triangle, square, regular pentagon, regular hexagon, and regular octagon tile the plane. Record your results in the table.

penta means 5
hexa means 6
octa means 8

5. Cut out the shape of an irregular quadrilateral.

a) Predict whether the shape will tile the plane.

b) Try to tile the plane with the shape. Record your results in the table.

c) Repeat steps 5a) and 5b) using an irregular pentagon and an irregular hexagon of your own design.

Reflect on Your Findings

6. a) What regular shapes tile the plane? Explain why some regular shapes tile the plane but others do not. Hint: Look at the interior angle measures. Is there a pattern?

b) Explain why some irregular shapes tile the plane but others do not.

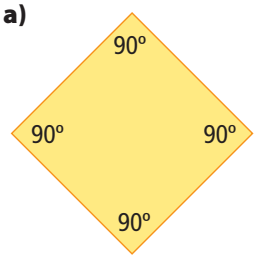
Literacy Link

The term *plane* means a two-dimensional flat surface that extends in all directions.

Example: Identify Shapes That Tessellate the Plane

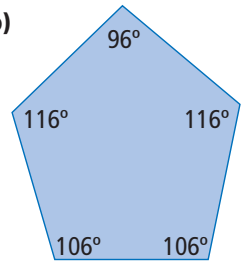
Do these polygons tessellate the plane? Explain why or why not.

a)



Shape A

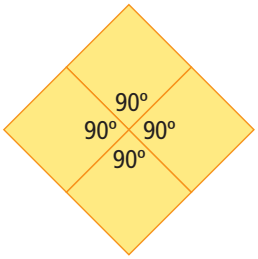
b)



Shape B

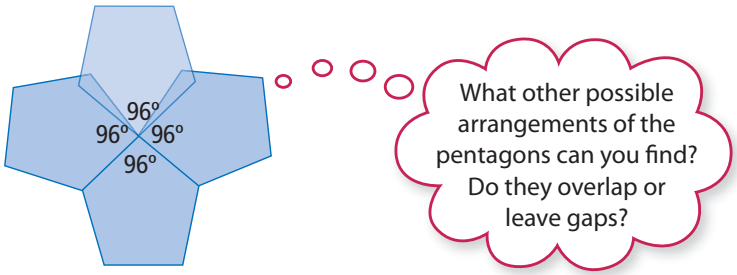
Solution

a) Arrange the squares along a common side. The rotated squares do not overlap or leave gaps when you try to form them into a tessellation. Shape A can be used to tessellate the plane.



Check:
Each of the interior angles where the vertices of the polygons meet is 90°. The sum of the four angles is $90° + 90° + 90° + 90° = 360°$. This is equal to a full turn. The shape can be used to tessellate the plane.

b) Arrange the pentagons along a common side. The irregular pentagons overlap or leave gaps when you try to form them into a tessellation. Shape B cannot be used to tessellate the plane.

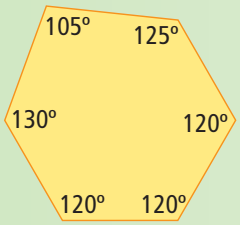


Check:
Each of the interior angles where the vertices of the polygons meet is 96°. The sum of the four angles is $96° + 96° + 96° + 96° = 384°$. This is more than a full turn. The shape cannot be used to tessellate the plane.

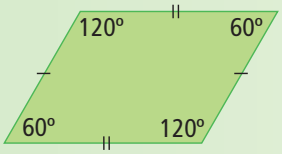
Show You Know

Which of the following shapes can be used to tessellate the plane? Explain your reasoning.

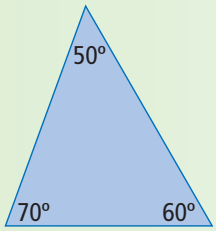
a)



b)

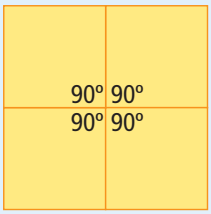


c)

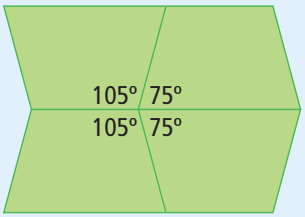


Key Ideas

- A tiling pattern or tessellation is a pattern that covers a plane without overlapping or leaving gaps.
- Only three types of regular polygons tessellate the plane.
- Some types of irregular polygons tessellate the plane.
- Regular and irregular polygons tessellate the plane when the interior angle measures total exactly 360° at the point where the vertices of the polygons meet.



$90° + 90° + 90° + 90° = 360°$



$105° + 75° + 75° + 105° = 360°$

Communicate the Ideas

1. Draw three types of regular polygons that tessellate the plane. Justify your choices.

2. What are two types of irregular polygons that can be used to tessellate the plane? Explain your choices to a friend.

3. Megan is tiling her kitchen floor. Should she choose ceramic tiles in the shape of a regular octagon? Explain how you know.

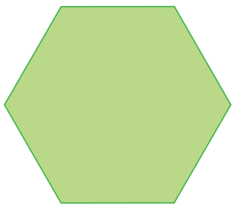
Check Your Understanding

Practise

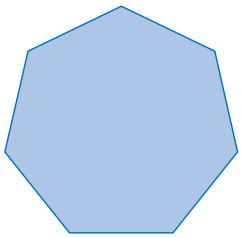
For help with #4 to #7, refer to the Example on page 448.

4. Do these regular polygons tessellate the plane? Explain why or why not.

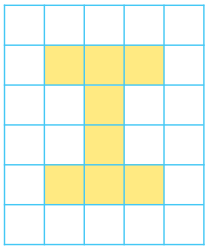
a)



b)



5. Use this shape to tessellate the plane. Show and colour the result on grid paper.



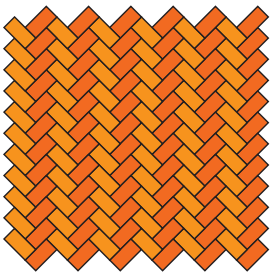
6. Tessellate the plane with an isosceles triangle. Use colours or shading to create an interesting design on grid paper.

7. Describe three tessellating patterns that you see at home or at school. What shapes make up the tessellation?

Apply

8. Jared is painting a mosaic on one wall of her bedroom that is made up of tessellating equilateral triangles. Describe two different tessellation patterns that Jared could use. Use triangular dot paper to help you describe the tessellations.

9. Patios are often made from interlocking rectangular bricks. The pattern shown below is called herringbone.



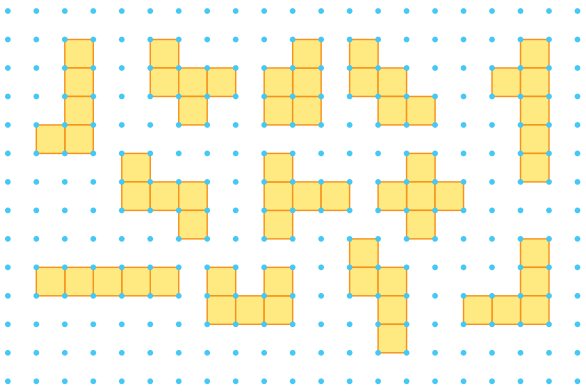
On grid paper, create two different patio designs from congruent rectangular bricks.

10. Some pentagons can be used to tessellate the plane.

a) Describe a pentagon that will tessellate the plane. Explain how it tessellates the plane.

b) Compare your pentagon with those of your classmates. How many different tessellating pentagons did you and your classmates find?

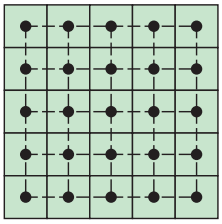
11. A pentomino is a shape made up of five squares. Choose two of the following pentominoes and try to make a tessellation with each one. Do each of your pentominoes make a tessellation? Explain why or why not.



12. Sarah is designing a pattern for the hood and cuffs of her new parka. She wants to use a regular polygon in the design and three different colours. Use grid paper to create two different designs that Sarah might use. Colour your designs.

Extend

13. The diagram shows a tessellation of squares. A dot has been added to the centre of each square. The dots are joined by dashed segments perpendicular to common sides. The result is another tessellation, which is called the *dual* of the original tessellation.



a) Describe the dual of the original square tessellation.

b) Draw a tessellation of regular hexagons. Draw and describe its dual.

c) Draw a tessellation of equilateral triangles. Draw and describe its dual.

14. Identify two different regular polygons that can be used together to create a tessellating pattern. Draw a tessellation on grid paper using the two polygons.

Did You Know?

Many Islamic artists make very intricate geometric decorations and are experts at tessellation art.

MATH LINK

This tiling pattern is from Alhambra, a Moorish palace built in Granada, Spain. Four different tile shapes are used to create this pattern.

a) Describe the four shapes. Are they regular or irregular polygons?

b) Use templates to trace the shapes onto cardboard or construction paper.

c) Cut out ten of each shape and use some or all of them to create at least two different tile mosaics. Use each of the four shapes in your mosaics.

WWW Web Link

To generate tessellations on the computer, go to www.mathlinks8.ca and follow the links.