10.1

Modelling and Solving One-Step Equations: $ax = b, \frac{x}{a} = b$

Focus on...

After this lesson, you will be able to...

- model problems with linear equations
- solve linear equations and show how you worked out the answer

When Simone tried her new pair of Moon Shoes, she wondered what made them so bouncy. She discovered that they have springs inside that store energy. How do you think this energy is used to make the shoes bounce?

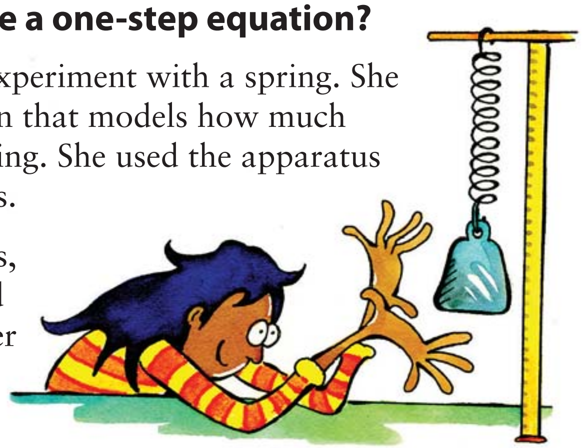
Explore the Math

Materials

- grid paper

How do you model and solve a one-step equation?

Simone decided to conduct an experiment with a spring. She wanted to determine an equation that models how much force is required to stretch a spring. She used the apparatus shown to take the measurements.



Every time Simone added a mass, the force on the spring increased and the spring stretched a further distance. The data that she collected during her experiment are shown in the table.

Trial	Force, *F* (newtons)	Distance Stretched, *d* (cm)
1	10	5
2	20	10
3	30	15
4	40	20
5	50	25

1. Draw a graph with Force on the horizontal axis and Distance Stretched on the vertical axis. Plot the values from the table.

2. a) How much more force is added for each trial?
   b) How much greater is the distance stretched each time force is added? Is the difference in the distance stretched the same for each consecutive trial?

3. What is the ratio, k, for the amount of force to the spring distance?

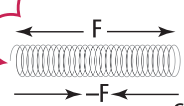
Reflect on Your Findings

4. What is a **linear equation** that models the relationship between force and distance stretched?

The force, F, required to stretch a spring distance, d, can be modelled using the equation $F = kd$. Use the value of k that you determined in #3.

5. a) If you use a force of 60 N, what is the distance the spring would stretch?
   b) How did you get your answer?

6. a) Imagine the spring is compressed instead of stretched. What would be the linear equation?
   b) How much force would it take to compress the spring 5 cm?



A positive force stretches a spring. A negative force compresses a spring.

Literacy Link

An *equation* is a mathematical statement with two expressions that have the same value. The two expressions are separated by an equal sign. For example,

$2x = 3$ $\quad \frac{a}{3} = 5$ $\quad b = 4$

In the equation $4y - 7 = -3$,
- the numerical coefficient is 4
- the variable is y
- the constants are 7 and −3

linear equation
- an equation that, when graphed, results in points that lie along a straight line
- examples are $y = 4x \quad d = \frac{c}{2}$
  $5w + 1 = t$

Example 1: Solve an Equation

Solve each equation.

a) $3x = -12$ $\qquad$ b) $\frac{r}{-2} = -7$

Solution

Method 1: Solve by Inspection

a) $3(-4) = -12$ $\qquad$ or $\qquad \frac{-12}{3} = -4$

Ask yourself, "What number multiplied by 3 equals −12?"

Ask yourself, "What number results from dividing −12 by 3?"

The solution is $x = -4$.

b) $\frac{14}{-2} = -7$ $\qquad$ or $\qquad -7 \times (-2) = 14$

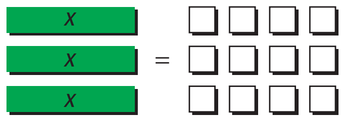
Ask yourself, "What number divided by −2 equals −7?"

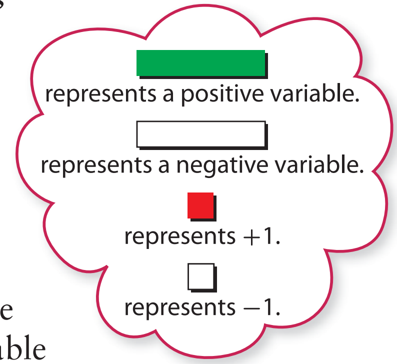
Ask yourself, "What number results from multiplying −7 by −2?"

The solution is $r = 14$.

Method 2: Solve Using Models and Diagrams

a) Use algebra tiles.

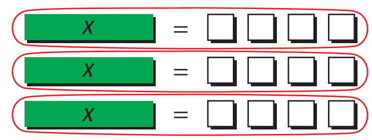




The three variable tiles represent $3x$.
The 12 negative 1-tiles represent -12.
The three variable tiles must have the same value as the 12 negative 1-tiles. Each variable tile must then have a value of four negative 1-tiles.

The solution is $x = -4$.

Check:



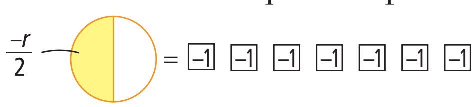
Left Side $= 3x$ Right Side $= -12$
$= 3(-4)$
$= -12$
Left Side = Right Side
The solution is correct.

b) Use a diagram.
Let one whole circle represent $-r$.

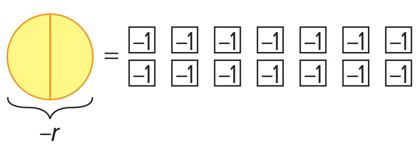
The expression $\frac{r}{-2}$ means the same as $\frac{-r}{2}$.

Then, one half of the circle represents $\frac{-r}{2}$ or $-r \div 2$.

The seven white squares represent -7.



Since half the circle represents $\frac{-r}{2}$, you need to double the shading or multiply by two to represent $-r$. To balance the equation, you need to double or multiply by two the number of white squares.



There are now 14 white squares representing -14.

What operation do you need to apply to both sides of the equation to solve for *r*?

So, $-r = -14$.
The solution is $r = 14$.

Check:

Left Side $= \frac{r}{-2}$ Right Side $= -7$

$= \frac{14}{-2}$

$= -7$

Left Side = Right Side
The solution is correct.

Show You Know

Solve each equation. Check your answer.

a) $-3t = -36$ **b)** $\frac{n}{3} = -7$

Example 2: Divide to Apply the Opposite Operation

Simone uses a different spring in her experiment. The equation that models this new spring is $F = 12d$, where F is the force, in newtons, needed to stretch or compress the spring a distance, d, in centimetres. Simone applies a force of 84 N to compress the spring. What distance is the spring compressed?

Literacy Link

An *opposite operation* "undoes" another operation. Examples of opposite operations are

- subtraction and addition
- multiplication and division

You may sometimes hear opposite operations called *inverse operations*.

Solution

Since Simone compressed the spring, the force, F, is a negative number. Substitute -84 into the formula $F = 12d$. Then, isolate the variable to solve the equation.

$$F = 12d$$
$$-84 = 12d$$
$$\frac{-84}{12} = \frac{12d}{12}$$
$$-7 = d$$

The opposite of multiplying by 12 is dividing by 12.

The spring was compressed a distance of 7 cm.

Check:

Left Side $= -84$ Right Side $= 12d$

$= 12(-7)$

$= -84$

Left Side = Right Side

The solution is correct.

Literacy Link

Isolate the variable means to get the variable by itself on one side of the equation.

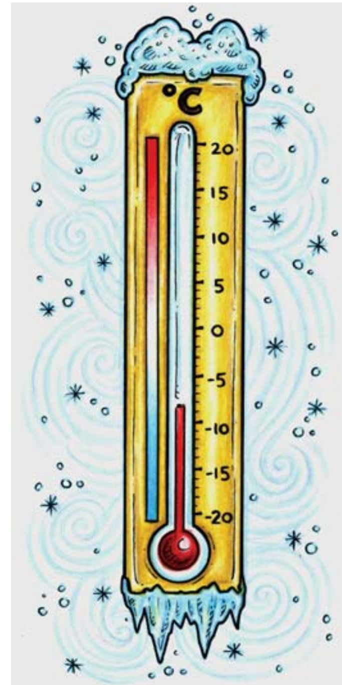
Show You Know

Solve by applying the opposite operation. Check your answer.

a) $-5b = -45$ **b)** $6f = -12$

Example 3: Multiply to Apply the Opposite Operation

For the month of January, the average afternoon temperature in Edmonton is $\frac{1}{3}$ the average afternoon temperature in Yellowknife. The average afternoon temperature in Edmonton is -8 °C. What is the average afternoon temperature in Yellowknife?



Solution

Let t represent the average afternoon temperature in Yellowknife.

The variable represents the unknown value.

The average afternoon temperature in Edmonton is $\frac{1}{3}$ the average afternoon temperature in Yellowknife, or $\frac{t}{3}$.

You can model the problem with the equation $\frac{t}{3} = -8$.

Solve the equation by applying the opposite operation.

Literacy Link

$\frac{1}{3}t$ is the same as $\frac{t}{3}$ or $t \div 3$.

$$\frac{t}{3} = -8$$
$$\frac{t}{3} \times 3 = -8 \times 3$$
$$t = -24$$

The opposite of dividing by 3 is multiplying by 3.

The average afternoon temperature in Yellowknife is -24 °C.

Check:

Left Side $= \frac{t}{3}$ $\quad$ Right Side $= -8$

$= \frac{-24}{3}$

$= -8$

The solution of -24 °C is correct.

Show You Know

Solve by applying the opposite operation. Check your answer.

a) $\frac{d}{-5} = 3$ **b)** $-6 = \frac{p}{7}$

Key Ideas

- There are several ways to solve equations involving integers.
  - Solve by inspection.

    $-2w = 6$

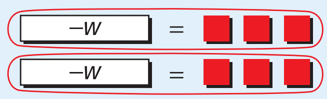
    $-2(-3) = 6$ or $\frac{6}{-2} = -3$

    Ask yourself, "What number multiplied by −2 equals 6?"

    Ask yourself, "What number results from dividing 6 by −2?"

    The solution is $w = -3$.
  - Model the equation using concrete materials and then balance it.

    $-2w = 6$



    Each negative variable tile must have a value of three positive 1-tiles.
    The positive variable tile must then have a value of three negative 1-tiles.
    The solution is $w = -3$.
  - Perform the opposite operation on both sides of the equal sign.

$$\frac{w}{-2} = 6$$

$$\frac{w}{-2} \times (-2) = 6 \times (-2)$$

$$w = -12$$

    The opposite of dividing by −2 is multiplying by −2.

- Two methods you can use to check your solution are substitution and modelling:
  - Substitute your solution into the equation. Both sides should have the same value.

    Left Side $= \frac{w}{-2}$ Right Side $= 6$

    $= \frac{-12}{-2}$

    $= 6$

    Left Side = Right Side

    The solution is correct.
  - Model the equation using concrete materials like algebra tiles as shown above.

Communicate the Ideas

1. Draw a diagram to show how you can model $\frac{x}{6} = -3$. Explain your diagram in words.

2. Give an example of an equation that has a variable with a negative integer value.

3. An unknown number is multiplied by 5. The result is -45.

a) Choose a variable. Write an equation to represent the situation.

b) Draw a picture to show how you might solve the equation.

4. Raj is solving the equation $\frac{n}{9} = -4$.

$$\frac{n}{9} = -4$$
$$\frac{n}{9} \times (-9) = -4 \times (-9)$$
$$n = 36$$

Is Raj's solution correct or incorrect? Explain.

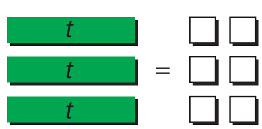
Check Your Understanding

Practise

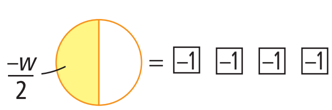
For help with #5 to #10, refer to Example 1 on pages 371–372.

5. Write the equation modelled by each diagram.

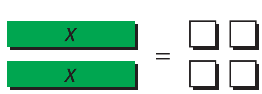
a)



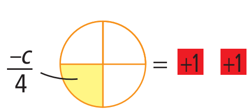
b)



c)



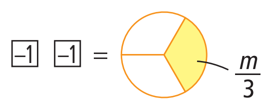
d)



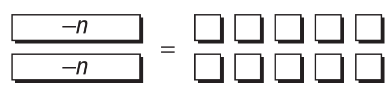
Recall that ■ represents +1 and □ represents −1.

6. Write the equation represented by each model.

a)



b)



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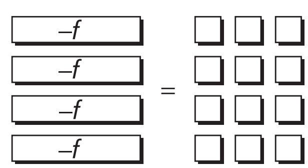
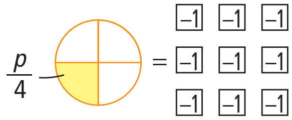


d)



7. Solve by inspection.

a) $-8j = 64$

b) $5n = -25$

c) $-6 = \frac{k}{3}$

d) $\frac{x}{-11} = -4$

8. Use mental math to solve each equation.

a) $-12 = 3r$

b) $-16 = -4p$

c) $-30 = \frac{t}{2}$

d) $\frac{d}{-4} = 5$

9. Use models or diagrams to solve each equation.

a) $2k = -8$

b) $-3 = \frac{t}{4}$

10. Solve each equation using models or diagrams.

a) $3b = -15$

b) $\frac{x}{-3} = -3$

For help with #11 to #14, refer to Example 2 on page 373.

11. By what number would you divide both sides of the equation to solve it?

a) $-3x = 9$

b) $-36 = -4g$

c) $72 = -9t$

d) $4p = -8$

12. By what number would you divide both sides of the equation to solve it?

a) $-10 = 5w$

b) $-48 = -4c$

c) $4y = -400$

d) $-84 = -21b$

13. Solve each equation using the opposite operation. Check your answer.

a) $4s = -12$

b) $-156 = -12j$

c) $-4j = 104$

d) $-108 = -27t$

14. Use the opposite operation to solve each equation. Verify your answer.

a) $8f = -56$

b) $-5q = 45$

c) $-2h = -42$

d) $14k = -70$

For help with #15 to #18, refer to Example 3 on page 374.

15. By what number would you multiply both sides of the equation to solve it?

a) $13 = \frac{g}{-6}$

b) $\frac{m}{3} = -25$

c) $-6 = \frac{n}{-21}$

d) $\frac{z}{17} = 6$

16. By what number would you multiply both sides of the equation to solve it?

a) $\frac{s}{11} = 9$

b) $-6 = \frac{y}{-12}$

c) $\frac{w}{4} = -13$

d) $16 = \frac{x}{-3}$

17. Solve each equation using the opposite operation. Check your answer.

a) $\frac{t}{3} = -12$

b) $12 = \frac{h}{-10}$

c) $\frac{s}{-7} = 15$

d) $-63 = \frac{x}{-9}$

18. Use the opposite operation to solve each equation. Verify your answer.

a) $\frac{y}{5} = -4$

b) $-6 = \frac{k}{-8}$

c) $-1 = \frac{b}{10}$

d) $\frac{r}{12} = 15$

Apply

19. Show whether $x = -2$ is the solution to each equation.

a) $-8x = 16$

b) $10x = -20$

c) $-5x = 10$

d) $36 = 18x$

20. Show whether $y = 12$ is the solution to each equation.

a) $3 = \frac{y}{-4}$

b) $\frac{y}{-36} = -3$

c) $2 = \frac{y}{24}$

d) $\frac{y}{-6} = -2$

21. For the month of January, the average afternoon temperature in Calgary is $\frac{1}{4}$ the average morning temperature. The average afternoon temperature is -4 °C. What is the average morning temperature?

a) If m represents the average morning temperature, what equation models this problem?

b) Solve the equation. Verify your answer.

22. Nakasuk's snowmobile can travel 13 km on a litre of gas. He is going to visit his aunt in a community 312 km away. Nakasuk wants to know how many litres of gas he needs to travel to his aunt's community.

a) Write an equation in the form $ax = b$ to represent this problem. What does your variable represent?

b) How many litres of gas does Nakasuk need?

23. The height of a great grey owl is five times the height of a pygmy owl. A great grey owl can grow to 85 cm.

Pygmy Owl

Great Grey Owl

a) Model this problem with an equation of the form $ax = b$. Tell what your variable represents.

b) What is the height of the pygmy owl?

24. Lucy is making four pairs of mitts. She has 144 cm of trim to sew around the cuffs of the mitts. How much trim does she have for each mitt?

a) Write an equation to represent this situation.

b) Solve the equation.

25. People can be left-handed, right-handed, or ambidextrous. The number of boys in Canadian secondary schools who are left-handed is about $\frac{1}{7}$ of the number of boys who are right-handed. About 11% of boys are left-handed. Write and solve an equation to determine what percent of boys are right-handed.

Did You Know?

There are more ambidextrous students in Canada than there are left-handed students. *Ambidextrous* means that you are able to use your left hand and right hand with equal ability.

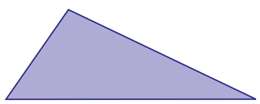
26. Kim works at an art gallery. An art dealer offers her a sculpture for \$36 000. The dealer says the current value of the sculpture is twice its value the previous year.

a) What was its value the previous year?

b) If the sculpture's value increases at the same rate next year, what will the new value be?

Extend

27. The area of the triangle shown is 30 cm^2. Write and solve an equation to determine its height.



12 cm

28. Workers are repairing a section of road that is 5 km long. The speed limit has been changed from 50 km/h to 20 km/h. How many minutes does this add to the drive along this section of road?

29. The formulas that give the length of time for sound to travel underwater are

$t = \dfrac{d}{149\ 700}$ for fresh water, and

$t = \dfrac{d}{150\ 000}$ for salt water, where t is time, in seconds, and d is distance, in centimetres.

a) If a sound travels for 2 s, what distance does it travel in metres in fresh water? in salt water?

b) Two scientists are doing an underwater study of dolphin sounds. Sandra is 90 cm away from a freshwater dolphin. Donald is 1 m away from a saltwater dolphin. Who hears each sound in less time, Sandra or Donald? Show your work.

MATH LINK

Have you ever dropped Silly Putty® onto a hard surface? It bounces! The greater the height from which a ball of Silly Putty® is dropped, the higher it bounces.

a) Design and perform an experiment that allows you to record how high a ball of Silly Putty® bounces when dropped from different heights.

b) Determine an equation that models the results of your experiment. Write the equation in the form $b = kh$, where h is the height from which the Silly Putty® ball is dropped, b is the height of the first bounce, and k is a numerical coefficient that you will determine from your experiment.

WWW Web Link

For a Silly Putty® recipe, go to www.mathlinks8.ca and follow the links.