## DETERMINING SURFACE AREA OF 3-D SHAPES

## Background Review:

$$
\text { RADIUS (abbreviated " } r \text { ") - }
$$

The distance from the center point of a circle to any point on the circle's perimeter


The radius of a circle is half of the length of the diameter, so $\boldsymbol{r}=\frac{d}{2}$
Ex. If the diameter of a circle is 6 cm , the radius would be 3 cm .

DIAMETER (abbreviated " $d$ ") - the length of a line segment that passes through the center of a circle dividing the circle into two equal halves.


The diameter of a circle is twice the length of the radius, so $\boldsymbol{d}=2 \times \boldsymbol{r}$
Ex. If the radius of a circle is 8 cm , the diameter would be 16 cm .

## CIRCUMFERENCE (abbreviated " $c$ ") - the entire distance around the outside of a circle


$\mathbf{P i}$, represented by the symbol " $\boldsymbol{\pi}$ ", represents how many times longer a circle's circumference is than its diameter (approximately 3.14 times).

Therefore, the length of a circle's circumference is 3.14 times longer than its diameter, so $\boldsymbol{c}=\boldsymbol{\pi} \times \boldsymbol{d}$

## Formulae:

$$
\begin{array}{lll}
r=\frac{d}{2} & d=2 r & c=\pi d \\
r=\frac{c}{2 \pi} & d=\frac{c}{\pi} & c=2 \pi r
\end{array}
$$

## Background Review (continued):

## AREA (abbreviated " $A$ ") - the amount of space covered by a two dimensional shape

SQUARE / RECTANGLE: The area of a square or rectangle is calculated by multiplying its length by its width, so:

$$
\boldsymbol{A}=\boldsymbol{l} \times \boldsymbol{w}
$$

For the rectangle below: $\quad A=l x w$ $A=6 \mathrm{~cm} \times 4 \mathrm{~cm}$ $A=24 \mathrm{~cm}^{2}$


TRIANGLE: The area of a triangle is always half of the area of a rectangle with the same sized base (or length in the rectangle) and the same sized height (or width in the rectangle). As a result, we can calculate the area of a triangle using the formula:
$\mathbf{A}=\frac{1}{2} \times \boldsymbol{b} \times \boldsymbol{h}$
Look at the triangle below. Its base is 12 m and its vertical height is 9 m . To calculate its area, we simply use the formula $\mathrm{A}=1 / 2 \mathrm{xb} \times \mathrm{h}$ to calculate its area:

$$
\begin{aligned}
& A=1 / 2 \times b \times h \\
& A=1 / 2 \times 12 m \times 9 m \\
& A=54 m^{2}
\end{aligned}
$$



$$
b=12 m
$$

In the example below, the base is marked ' $b$ ' and the vertical height is marked ' $h$ '. We use the same formula to calculate the area of this triangle ( $\mathrm{A}=1 / 2 \times \mathrm{x} \times \mathrm{h}$ )


A $=1 / 2 \times \mathbf{x} \quad \mathbf{x} h$
$A=1 / 2 \times 8 \times 5$
$A=20$ square units

NOTE: In both triangles above, I have added light grey lines to show that triangles are indeed ALWAYS HALF OF A RECTANGLE.

CIRCLE: The area of a circle is calculated by multiplying Pi by the square of the length of the circle's radius, so:
$\boldsymbol{A}=\pi \times \boldsymbol{r}^{2} \quad$ So for the circle below:
$\mathbf{A}=\pi \times r^{2}$
$\mathbf{A}=3.14 \times(3 \mathrm{~cm})^{2}$
$\mathbf{A}=3.14 \times 9 \mathrm{~cm}^{2}$
$\mathbf{A}=28.26 \mathrm{~cm}^{2}$

## Calculating Surface Area of a 3-D Object:

## SURFACE AREA (abbreviated "SA") - the area of the outermost part of a three-dimensional figure

To calculate the surface area of a three-dimensional object, one must simply calculate the area of each outer face or surface and add these areas together. In Math 8, we must be able to calculate the surface area of a rectangular prism (Figure - 1), a cylinder (Figure - 2), and a triangular prism (Figure - 3).

(FIGURE - 1)

(FIGURE - 2)

(FIGURE - 3)

One way to ensure that we calculate the area of EVERY surface on our three-dimensional shape, before we add these areas up, is to sketch a 'net' of each shape. A net is a two-dimensional sketch of all surfaces of a 3-D shape if each was unfolded and laid flat. Below is a sketch of the net for each of the 3D shapes above:


We should be able to see that diagram ' $A$ ' is the net for the 3-D shape in Figure - 1, that diagram ' $B$ ' is the net from the shape in Figure - 2, and that diagram ' C ' is roughly the net for the shape in Figure -3 . Looking at these nets, we can see that the surfaces of the three-dimensional shapes above are made up of triangles, rectangles and circles - all shapes for which we are able to calculate their areas. Therefore, to calculate the total surface area of any of these shapes, we simply have to add up the areas of each shape in the net.

IMPORTANT SHORTCUT:

If we look closely at the nets of the figures above, we notice that some of the shapes in each net have the same measurements. In the rectangular prism, there are three pairs of identical sides. In the cylinder, the two circles are the same size. In the triangular prism, the two triangles are the same size. Therefore, we do not have to calculate the area of ALL sides, if there are any equal sides, calculate the area of one, and double its value when calculating surface area.

